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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

A. W. WUNDHEILER, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ĭ	′	è	yu	ya

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

May 1950 Vol. 3, No. 5

Communications

Reviewers wanted

APPLIED MECHANICS REVIEWS does not have specialized reviewers for papers in:

Russian, on acoustics;

Italian, on heat conduction, heat transfer, and general theory of dynamical systems.

We appeal to our readers for offers of their services or suggestions as to names. Ed.

Concerning Rev. 80, January 1950, "Plastic flow of concrete relieves high-load stress concentrations," by Robert F. Blanks and Douglas McHenry.

It appears to me that the reviewer's statement: "When the concrete is one year old, however, the tests show a pronounced difference in the loading capacity of the solid and hollow blocks," is at variance with the data presented by the authors. Column six of Table 1 shows the unit strength of the one-year old pierced blocks averaged 108.3% of that of the solid blocks, which is slightly better than shown for the 28-day old blocks. It would thus seem that where this test data can be applied, the stress concentrations produced by the holes need not be considered except in connection with the formation of initial cracks.

A. D. Hogg, Toronto

As pointed out by Mr. A. D. Hogg, the statement, "When the concrete is one year old the tests show a pronounced difference in the loading capacity of the solid and hollow blocks," is at variance with the data presented in Blanks and McHenry's paper. The fact the reviewer wanted to bring out was that the load at initial observed cracking for blocks tested at the age of one year is only one-half of the cracking load for blocks tested at 28 days. According to the tests this is true for solid blocks as well as for hollow blocks, in spite of the fact that the ultimate load about doubles in the same time for both types of blocks. This seems to suggest that either the ultimate tensile stress decreases with time, which is not likely, or that the material increases its tensile notch sensitivity with time, which should have important consequences in design. The reviewer also thinks that consideration should be given to the load-carrying capacity of concrete specimens when the load acts on the specimens for long periods of time, as is the case in the actual structures. A. J. Durelli, Chicago

Theoretical and Experimental Methods

(See also Revs. 828, 884, 933, 934, 935, 977)

806. A. E. Heath et al., The impact of science on society, Nature Lond. 164, no. 4170, 555–557 (1949).

This article is a résumé of the discussions on the general subject of "Science and Society" given at the fourth annual conference of the Rationalist Press Association held at Magdalen College, Oxford. The topics discussed were: (1) the application of the scientific method for the solution of problems in the field of the social sciences, by Prof. A. E. Heath; (2) the impact of scientific developments upon society in so far as the peace of the world is concerned, by Lord Boyd Orr; (3) the value of science in modern civilization as distinct from its technical applications, by Dr. Meyer Fortes; (4) the need for including the study of hygiene in the general scientific education, by Sir Sheldon Dudley; (5) the control of population compatible with economic conditions, by Dr. C. P. Blacker. Freeman K. Hill, USA

807. Louis A. Robb, Engineers' dictionary: Spanish-English and English-Spanish, second edition, New York: John Wiley and Sons; London: Chapman and Hall, 1949, xvi + 664 pp. Cloth, 6 × 8.5 in., $12.50.

This edition brings up to date the terms in civil engineering (photogrammetry, soil mechanics, and airport construction), mining, shipbuilding, logging, sugar milling, and oil-field operations. Marginal terms (chemistry, geology, metallurgy, finance, insurance, transportation) are selectively included. Special attention is given to phrases ("buttress thread"). Definitions are left out deliberately. Established Spanish equivalents of proprietary trade names are also listed. The gender of all Spanish nouns is indicated. Ed.

808. G. W. Spenceley and R. M. Spenceley, Smithsonian elliptic functions tables, Washington, D. C.: Smithsonian Miscellaneous Collections, vol. 109 (Publication 3863), The Smithsonian Institution, 1947, iv + 366 pp., $4.50.

The book contains tables of functions sn u, cn u, dn u, $E(\phi,k)$, $A(r)$ and $D(r)$. The arguments θ and r take values $1°, 2°, \ldots, 90°$. The values of the functions have been computed with 15 decimal figures, the maximum divergence being ±4 in the 15th decimal. In printing, they have been cut to 12 decimal figures; the tables assure the accuracy of the 12th decimal digit. The values of K, K', E, E', D(90), 1/(D90) are given for each $\theta = 1°,2°, \ldots,90°$ with 15 decimals, q and q' with 16 correct decimals. In the appendix there are short tables of hyperbolic functions with 15 decimals for $r = 1°,2°, \ldots, 90°$.

Courtesy of Mathematical Reviews P. J. Myrberg, Finland

809. P. E. Krasnushkin, On the asymptotic representation of solutions of the wave equation (in Russian), Vestnik Moskov. Univ. 3, no. 6, 73–76 (1948).

In the equation (1) $d^2\phi/dz^2 + W(z)\phi = 0$, let $W(z)$ be an analytic function of z in which z occurs only in the combination $\epsilon^{1/2}z$, where ϵ is a small positive parameter. In order to solve (1) asymptotically for small ϵ the author transforms (1) into a simpler equation

$$d^2\psi/d\zeta^2 + P(\zeta)\psi = 0$$

by setting $\phi = f^{1/2}\psi$, $dz/d\zeta = f$. The problem of finding $f(z)$ when $P(\zeta)$ is prescribed is solved formally by writing f^2 and ζ as power

series in ϵ with indeterminate coefficients that are functions of $\epsilon^{1/2}z$. Then ϕ is calculated approximately by using only the first terms of these series. No complete discussion of the validity of this procedure is given, but conditions are formulated which guarantee the smallness of the second term in the resulting expression for ϕ. A physical interpretation in terms of wave propagation in nonhomogeneous media is added.

Courtesy of Mathematical Reviews W. Wasow, USA

810. Raymond Redheffer, Errors in simultaneous linear equations, Quart. appl. Math. 6, 342–343 (1948).

Let the true solutions be x_i, the computed solutions x'_i, so that $\Sigma a_{ij}x_i = b_i, \Sigma a_{ij}x'_i = b'_i$. Then the author finds the following relation between the errors of the unknowns and the errors of the right sides: $E \leq e/\lambda^{1/2}$, where λ is the minimum characteristic root of AA'; $E^2 = \Sigma(x_i - x'_i)^2$, $e^2 = \Sigma(b_i - b'_i)^2$. From this relation it follows that

$$E < e(T/\nu)^{\nu/2}/|\det A|,$$

where

$$\nu = n - 1, \; T = \Sigma\Sigma a^2_{ij} = \text{norm of A}.$$

Courtesy of Mathematical Reviews E. Bodewig, Holland

811. L. A. Lyusternik and V. A. Ditkin, The construction of approximate formulas for the calculation of multiple integrals (in Russian), Doklady Akad. Nauk SSSR 61, 441–444 (1948).

812. Dale R. Branchflower, A method of least square adjustment for measured wing deflections, J. aero. Sci. 16, 375–376 and 383 (June 1949).

The dynamic characteristics of an airplane wing are predicted from calculations based on the theoretical bending-torsion modes of the wing. The validity of these predictions depends upon experimental verification of the theoretical modes. For this purpose measured wing deflections are obtained from ground vibration tests.

This paper applies the principles of least-square adjustment to the measured modes in such a manner that a best fit is obtained for the kinetic energy. Furthermore, the measured deflections are expressed as a series of theoretical deflections. Probably the most important feature of the method given is that the coefficients in the series are a numerical measure of the similarity of theoretical and experimental modes. Author's summary

813. B. Woolf, Rapid calculation of standard deviations, Nature Lond. 164, no. 4165, 360–361 (1949).

A graphical method is presented, based on the Pythagorean theorem, for calculating the square root of a sum of squares. By using the relation $2\Sigma xy = \Sigma x^2 + \Sigma y^2 - \Sigma(x - y)^2$, a similar method is suggested for calculating a sum of products.

Nils Blomqvist, Sweden

814. G. S. Smith, Rapid calculation of standard deviations, Nature Lond. 164, no. 4173 (1949).

A simplification of a method suggested by B. Woolf (see preceding review). Nils Blomqvist, Sweden

815. A. W. Joseph, A comment on interpolation in two variables, J. Inst. Actuar. 74, 82–85 (1948).

The author gives a geometric interpretation of the usual elementary method of interpolation in two variables in which a polynomial of specified degree in x and y together is fitted to values of $f(x, y)$. He shows that the number of given values $f(x_i, y_i)$ required can be reduced somewhat by choosing the points (x_i, y_i) so that a number of them lie on the same curve (of the specified, or lower, degree) with the point (x, y) for which an interpolated value is desired.

Courtesy of Mathematical Reviews T. N. E. Greville, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Rev. 914)

816. S. Trailescu, On the second order acceleration (in French), Bull. sci. Éc. Polyt. Timişoara no. 3, 13, 239–245 (1948).

In this reviewer's opinion the first part is interesting but incorrect, while the second is correct but uninteresting.

Gino Moretti, Argentina

817. E. Figueras Calsina, Distribution of superaccelerations in a spherical rigid motion (in Spanish), Revista Mat. Hisp.-Amer. 8, 155–164 (1948).

As it is used in this paper, the term superacceleration (of a point) means the time-rate of change of the acceleration of the point. The author considers a rigid spherical shell moving arbitrarily in such a way that the center of the shell remains fixed, and he studies the distribution of the superaccelerations of the points of the shell at an arbitrary instant. Various formulas and theorems are given, relating to the superaccelerations and their components. One of the simplest of the results is the following: If there exist points for which the instantaneous superacceleration vanishes, these are either (1) just two diametrically opposite points, (2) the points of a certain circle, or (3) all of the points of the shell.

Courtesy of Mathematical Reviews L. A. MacColl, USA

818. Bekir Dizioglu, Special R_M curves and their applications (in German, with Turkish summary), Rev. Fac. Sci. Univ. Istanbul (A) 14, 49–64 (1949).

Let a movable plane body assume three different positions in the fixed plane. Let the radius of circle through corresponding points be R. The R_M-curves, which were extensively investigated by H. Alt [Z. angew. Math. Mech. **1**, 373–398 (1921)] and A. E. Mayer [Math. Z. 43, 389–445 (1937)], are the loci of points which have the same R. These curves are tricircular sextics and are special cases of the "three-bar curves" (*Koppelkurven*) traced by a point on the connecting rod joining two cranks. The triangle of the double points coincides with the triangle of the focal points. Special cases are investigated here including those in which this triangle is equilateral and the three-bar curve has three cusps. Constructions are given for points on the R_M-curve. These are applied to the determination of the linkage which makes four given positions of a link correspond to four given positions of a crank.

M. Goldberg, USA

819. K.-H. Sieker, Determination of four-bar linkages from the radii of curvature of the centrodes and their derivatives (in German), Technik 3, 170–174 (Apr. 1948).

To solve the title problem, the author first combines certain known geometric relations for the radii of curvature of the fixed and moving centrodes (pertaining to the floating link), the inflection-circle diameter, and the derivatives of these distances, with respect to centrode-arc length. The solution of the synthesis problem is presented, in which the linkage dimensions are calculated from prescribed values of the above quantities. The author then proceeds to the more important, inverse problem in which, with the centers of driver and follower prescribed, a linkage is to

be determined which transmits a given angular displacement relation $\psi = f(\varphi)$ between these two links. For this purpose, he derives formulas expressing the radii of curvature of the relative centrodes of driver and follower in terms of ψ and its derivatives. It is found that a four-bar linkage can be determined which follows ψ and its first four derivatives exactly at one position, and, consequently, approximates ψ well in the neighborhood of this position. Two numerical examples for such a problem are solved in detail. The nature of the method makes it applicable to problems in which ψ is to be approximated accurately at and near one position. It is not directly applicable to synthesis problems in which ψ is to be approximated over a complete motion cycle.

G. A. Nothmann, USA

820. **G. Henriot, A treatise on the theory and practice of gears, vol. II (Traité théorique et pratique des engrenages),** Paris, Dunod, 1950, viii + 335 pp., 711 figs. Cloth, 9.7 × 6.3 in., $4.93.

This second volume (see Rev. 3, **203**) is devoted to the practice of manufacturing gears. It serves as a handbook of methods, tools and machines, thoroughly illustrated with photographs, line sketches and gearing diagrams. The three principal methods are: cutting by reproduction, cutting with a form cutter, and cutting by generation.

Most of the book is devoted to cutting by generation. The use of rack tools for making spur gears and splines is described and illustrated in various machines. The cutting of straight and bevel gears by the Bilgram system with flat tools, the generation of the Gleason type of spiral bevel gears (including zero pitch and hypoid gears) by the use of offset rotary cutters, the cutting of formate gears by the spread-blade method, the cutting of straight worms, global or Hindley worms, and the cutting of worm wheels are illustrated in various machines. Among the commercial lines described are the Citroën, Sunderland, Precimo, Barber-Coleman, Lees-Bradner, Fellows, Sykes, Shear Speed, Reinecker, Gleason, Paul Durand, Pratt and Whitney, and Maag.

Finish grinding by flat, conical, biconical and shaped grinders is illustrated. The grinding machines described include Maag, Niles, Pratt and Whitney, Reishauer, Sfedr and Minerva machines. Shaving machines, which have practically replaced grinders in the United States, are illustrated in the Red Wing machines. Fellows and Michigan Tool make similar machines.

The methods of preparation and sharpening of cutting tools by jobbing and machine methods and the machines for performing these operations are explained. The dressing of grinding wheels, including the forming of shaped wheels, is described.

The qualities of gear production are classified by the errors of pitch, eccentricity, errors of profile, distortion and state of the surface. The methods of measuring these, both manual and automatic, are described.

Heat treatments and chemical treatments are described for improving the strength, hardness and wearing qualities of the various materials. Machines for the automatic control of these processes are described. Assemblies of gears like speed reducers and gear pumps and the problems of wearing-in, lapping, polishing and lubrication are discussed. Michael Goldberg, USA

821. **Edward Kasner and John De Cicco, Geometric theorems in dynamics,** Math. Mag., 223–233 (1949).

This paper deals with a problem which has been treated by Kasner in previous papers [e.g., Trans. Amer. math. Soc. 7, 401–424 (1906)] and in his monograph, *Differential-geometric aspects of dynamics* [Amer. math. Soc. Colloquium Publ. vol. 3, part 2, New York, 1913]. Specifically, the problem is that of determining a set of properties which is necessary and sufficient for a three-parameter family of plane curves to be the family of trajectories of a particle moving in a positional field of force. This paper gives new proofs of some previously known theorems, and elaborates further on various aspects of the problem.

Courtesy of Mathematical Reviews L. A. MacColl, USA

822. **Homer V. Craig, On extensors and the Lagrangian equations of motion,** Math. Mag. 22, 255–251 (1949).

A derivation is presented of Lagrange's equations of motion of a conservative system based on certain assumptions on invariance and dependence on the kinetic and potential energy functions.

Courtesy of Mathematical Reviews P. Franklin, USA

823. **F. A. Valentine, The motion of a sliding horizontal hoop,** Amer. Math. Monthly 56, 79–87 (1949).

By analytical reasoning on inequalities and comparison equations, qualitative results are deduced for a problem in mechanics whose equations do not admit of an elementary solution.

Courtesy of Mathematical Reviews P. Franklin, USA

824. **Luca Teodoriu, Stability of equilibrium of a material point** (in French), Disquisit. Math. Phys. 6, 137–191, 5 plates (1948).

A study of the imperfect equivalence of dynamic stability and static stability, or the sign of the work done in a virtual displacement, is presented. The systems considered are assumed to be represented by their osculating systems of linear equations with constant coefficients, and lengthy calculations are made for a few typical systems of this type.

Courtesy of Mathematical Reviews P. Franklin, USA

825. **Jules Haag, On the existence and stability of the periodic solutions of certain systems of differential equations** (in French), Ann. Sci. École Norm. Sup. 65, 299–335 (1948).

A study of the existence and stability of periodic solutions of a system of differential equations $\dot{x}_i = f_i(x,t)$ is given. The author begins with linear equations, and extends his results to nonlinear equations by the classical perturbation method. The conjecture on page 315 that the absolute value of the characteristic roots of the variational equation must be less than one for stability is not true.

Courtesy of Mathematical Reviews F. Bohnenblust, USA

826. **N. P. Erugin, A generalization of a theorem of Liapounoff** (in Russian), Prikl. Mat. Mekh. 12, no. 5, 632–638 (1948).

The solutions of $x'' + p(t)x = 0$, where $p(t)$ is a periodic of period 1 and $p(t) = 0$, are stable if $\int_0^1 p(t)dt = 4$ according to a well-known theorem of Liapounoff. By analyzing the proof of Liapounoff in greater detail the author derives other criteria for stability. N. Levinson, USA

827. **A. P. Gremyachenskiĭ, A generalization of a theorem of Liapounoff** (in Russian), Prikl. Mat. Mekh. 12, no. 5, 667–668 (1948).

A theorem of Liapounoff states that the system of differential equations $dx/dt = P(t)x$, where x is a vector and $P(t)$ is a periodic matrix, is solvable by n quadratures if there exists a matrix B' with constant coefficients such that $BP(t) + P(-t)B = 0$ and such that the characteristic roots of B are all distinct as are their squares. The author shows that the matrix B may have repeated roots providing there is at most one independent vector solution v to the equation $(B - \lambda)v = 0$ for any fixed λ.

N. Levinson, USA

828. M. I. Almukhamedov, On conditions for the existence of stable and unstable centers (in Russian), Doklady Akad. Nauk SSSR 67, no. 6, 961–964 (1949).

This paper contains various necessary and sufficient conditions for a center for the differential equation in polar coordinates

$$\frac{d\rho}{d\omega} = \frac{\rho^2\varphi_1 + \rho^3\varphi_2 + \ldots + \rho^{n+1}\varphi_n}{1 + \rho\psi_1 + \rho^2\psi_2 + \ldots + \rho^n\psi_n} \tag{1}$$

where ρ_s, ψ_s are polynomials in $\sin\omega, \cos\omega$. In particular one may reduce to (1) the equation

$$\frac{dy}{dx} = -\frac{x + P_n(x,y)}{y + Q_n(x,y)} \tag{2}$$

where P_n, Q_n are polynomials of degree n beginning with terms of degree at least two. It is well known that for (1) the origin is a focus or a center. To distinguish between them the author endeavors to find a closed curve,

$$r + r^2\theta_1 + \ldots + r^{m+1}\theta_m = \epsilon,$$

where r is the polar distance, the θ_s polynomials in $\sin\omega, \cos\omega$, ϵ a small positive constant, and where along γ the difference

$$P'(\omega) - r'(\omega)$$

has a fixed sign. Thus γ is a curve without contact in the sense of Poincaré. If a suitable curve γ exists the origin is a focus; otherwise it is a center. Furthermore the number of conditions necessary to determine which of the two occurs is finite. The author's results include as special cases those of Poincaré [*Oeuvres*, I, p. 95] and Liapounoff [1892 Russian mémoire; French translation reprodu ed as Ann. of Math. Studies no. 17].

S. Lefschetz, USA

Gyroscopics, Governors, Servos

(See also Rev. 963)

829. A. I. Lur'e, On a canonical form of the equations of the theory of automatic regulation (in Russian), Akad. Nauk SSSR Prikl. Mat. Mekh. 12, 651–666 (1948).

The system

$$y_k = \sum_{l=1}^{n} b_{kl}y_l + h_k f(s),\ s = \sum_{l=1}^{n} j_l y_l,\ k = 1, \ldots, n,$$

is put into the canonical form

$$\dot{x}_i = \lambda_i x_i + f(s),\ \dot{s} = \sum_{l=1}^{n} \beta_l x_l - rf(s),\ i = 1, \ldots, n,$$

by means of a linear transformation $x_i = {}_1\Sigma_1{}^n c_l{}^i y_l$, in the case where the matrix $B = (b_{kl})$ has simple characteristic roots $\lambda_1, \ldots, \lambda_n$. Formulas are given for the constants $c_l{}^i, \beta_l, r$. Some physical examples are considered.

Courtesy of Mathematical Reviews J. G. Wendel, USA

Vibrations, Balancing

(See also Revs. 828, 829, 893, 922)

830. Karl Willy Wagner, Introduction to the theory of vibrations and waves (Einführung in die Lehre von den Schwingungen und Wellen), Wiesbaden, Dietrich, 1947, xvi + 640 pp., 288 figs. 6 × 9 in., approx. $8.

In spite of its bulk, this is only a first text in vibrations of discrete and continuous systems, mechanical and electrical, for the use of the research engineer. Only basic mathematics is assumed (appendixes present matrices and the Laplace transform). The basic problems are treated in much detail, and closed formulas are the goal of the solutions. Stability, the structure of the phase space and automatic regulation are not taken up at all, while nonlinear systems are given 87 pages, and are devoted essentially to examples. The volume of the book derives from the multitude of problems, and the detail of their treatment.

Only the mechanical topics will be enumerated here. Chapter I (100 pp.) treats the kinematics of vibrations and vibrating waves, and presents the basic elements of Fourier series. Simple transients and modulation are included. Chapter II (96 pp.) takes up the dynamical problems in one-dimensional vibrations, and coupled systems of finite degree of freedom. Chapter III (230 pp.) presents vibrations of continuous systems (ten kinds of strings in transversal vibrations; longitudinal and axial vibrations of beams and shafts; air columns; flexural vibrations of beams; rectangular and circular membranes; air in a rectangular room; spherical waves). Electrical networks claim 78 pages in this chapter.

Chapters IV and V (47 pp.) take up electric waves. Chapter VI (87 pp.) deals with some nonlinear systems: variable spring coefficient is the only mechanical one among them.

Those looking for a quick and painless survey of vibration phenomena will be satisfied with this unassuming book. (It is a photographic reproduction of the original 1943 edition.)

A. W. Wundheiler, USA

831. Fritz Söchting, Free and forced damped vibrations with nonlinear characteristics of a system with one degree of freedom (in German), Öst. Ingen.-Arch. 1, 382–389 (1947).

The equation of the pendulum, and some other ones, is treated by an approximation method related to Rayleigh's. Comparison is made with conventional solutions. Ed.

832. Dana Young and Robert P. Felgar, Jr., Tables of characteristic functions representing normal modes of vibration of a beam, Univ. Texas publ. no. 4913, 31 pp. (1949).

The characteristic function and its first three derivatives are tabulated numerically for each of the first five normal modes of a uniform beam with the ends clamped-clamped, clamped-free, or clamped-supported. To the bibliography should be added K. Hohenemser and W. Prager, *Dynamik der Stabwerke*, Berlin, 1933, and I. V. Anan'ev, *Reference book for computations of inherent vibrations of elastic systems* (in Russian), Moscow, 1946.

Robert D. Specht, USA

833. A. M. Sengupta, Note on a simple case of forced torsional oscillation of a circular cylinder, Bull. Calcutta Math. Soc. 41, 47–48 (1948).

One end of a circular elastic cylinder is fixed, the cylindrical surface is free from stress, and to the other end is applied a shear fraction proportional to the distance from the axis and trigonometric in time. The displacement is given in terms of a double trigonometric series. G. F. Carrier, USA

834. Morris Morduchow, On internal damping of rotating beams, Nat. adv. Comm. Aero. tech. Note no. 1996, 14 pp. (1949).

Based on the assumption that the structural damping force is proportional to the product of the time-rate of change of the elastic load, the internal damping coefficient of the beam, and a factor chosen so that the logarithmic decrement of a stationary beam oscillating harmonically is independent of frequency, the author shows that in general, rotation of a beam decreases the

structural logarithmic decrements of the beam, the effects normally being more pronounced for the lower modes of vibration.
Benjamin Smilg, USA

835. O. Föppl, Basic diagram for the determination of critical speeds of shafts with several masses attached (in German), Technik 3, 253–256 (June 1948).

This paper is essentially a brief treatment of the problem of the critical vibration frequencies of two equal masses symmetrically attached between bearings to a rotating shaft.
W. H. Hoppmann, II, USA

836. J. Dick, The whirling of shafts having sections with unequal principal bending moduli, Phil. Mag. 39, 946–955 (Dec. 1948).

The problem is approached by considering the equilibrium of forces along the principal bending axes, and simplified by assuming that the shaft is carrying a disk whose weight is large compared with the weight of the shaft. It is shown that there are two main whirling speeds corresponding to the frequencies of transverse vibrations along the two principal axes. With a shaft rotating about a nonvertical axis there is a whirl induced by gravity. This whirl ω occurs at twice the shaft speed and can be expressed as a function of the two main critical whirling speeds ω_1 and ω_2 by the relation

$$\omega = \omega_1\omega_2/[2(\omega_1^2 + \omega_2^2)]^{1/2}.$$

Free vibrations can occur in elliptical orbits having axes on the principal bending axes. When the speed is greater than ω_2 or less than ω_1, there are two possible orbits, and therefore two possible frequencies. If the speed lies between ω_1 and ω_2, there is instability. The effects of damping is also considered, and it is shown that it slightly changes the values of the critical speeds. Its effect is to reduce the amplitudes of whirls and the range in which instability occurs. Enrico Volterra, USA

837. J. O. Hinze, Effect of cylinder-pressure rise on engine vibrations, Ingenieur 's-Gravenhage 61, no. 48, 75–82 (1949).

In this paper the author compares the vibration of an engine to that of a simple spring mass system. From this he draws the conclusion that the ratio between the duration of rapid pressure rise to the lowest natural period of engine vibration is the primary factor if this ratio is above 0.25. Below 0.25 this ratio is only of secondary importance and the rapidity of pressure drop after maximum becomes of primary importance. The author claims that these criteria explain all the apparent discrepancies in the results obtained by numerous earlier investigators.
N. O. Myklestad, USA

Wave Motion, Impact

(See Revs. 861, 911, 965)

Elasticity Theory

(See also Revs. 833, 856, 875, 887, 897, 898, 899)

838. H. Richter, The isotropic elasticity law (in German), Z. angew. Math. Mech. 28, 205–209 (July-Aug. 1948).

The paper deals with possible elasticity laws for isotropic materials admitting large deformations, and taking changes of temperature into account. If P is the stress tensor and A the matrix of the deformation with respect to a stress-free state at a value Θ_0 of the temperature Θ, then the material is called elastic if P depends only on A and Θ, and isotropic if this relationship is invariant under Euclidean rotations. As each A can be represented by a rotation R followed by a straining S, isotropy requires that P be a function of S and Θ only. Instead of S the matrix $L = \log S$ may be used with advantage. The most general relationship between P and L or S and Θ is established that obeys the requirements of the isotropy and the thermodynamic potentials. The use of L instead of S has not only the advantage of a simpler formulation of the elasticity law, but also that the resolution of a deformation into a change of volume and one of shape is simply done by deviator forming, just as with infinitesimal deformations. It is shown that the elastic energy is then precisely equal to the sum of the energy of change of volume and the energy of change of shape if the average stress depends on the change of volume only. This is moreover independent of the choice of the reference temperature Θ_0. Hooke's law is shown to be compatible with the formula found for isothermic large deformations if, and only if, the Lamé constants satisfy $\lambda = 2\mu$, what corresponds to a Poisson ratio 1/3. In other cases a corresponding logarithmic law may be used. A. van Wijngaarden, Holland

839. C. Weber, Stress functions in a three-dimensional medium (in German), Z. angew. Math. Mech. 28, 193–197 (July-Aug. 1948).

Two sets of stress functions F_{ij} $(i,j = 1,2,3)$ which arise from the stress equations of equilibrium without body forces, are investigated. These functions are related to those given by Love [*The mathematical theory of elasticity*, 4th ed., Dover Publications, N. Y., 1944] as follows:

$$F_{11} = \chi_1,\ F_{22} = \chi_2,\ F_{33} = \chi_3,$$
$$F_{12} = -\tfrac{1}{2}\psi_3,\ F_{23} = -\tfrac{1}{2}\psi_1,\ F_{31} = -\tfrac{1}{2}\psi_2.$$

It is shown that if the stresses τ_{gh} are expressed in terms of any set of stress functions in the form

$$\tau_{gh} = \sum_{i,k} \partial^2 F_{ik}^{(gh)}/\partial x_i\,\partial x_k,$$

these stress functions are essentially the same as those given above. The array of stress functions F_{ij}, where $F_{ij} = F_{ji}$, constitutes a tensor. Furthermore, the differences of any two sets of stress functions, which yield the same state of stress, satisfy the compatibility equations for small strains. [Reviewer's note: the last result is indicated briefly in the section in Love cited above.] G. H. Handelman, USA

840. Daniel Rachkovitch, The potential of an elastic body in dyadic form (in French, with Turkish summary), Acad. Serbe Sci. Publ. Inst. Math. 1, 136–142 (1947).

841. Daniel Rachkovitch, Fundamental equations of elasticity in dyadic form (in French, with Turkish summary), Acad. Serbe Sci. Publ. Inst. Math. 2, 248–256 (1948).

842. Dario Graffi, On the reciprocity theorem in the dynamics of elastic bodies (in Italian), Mem. Accad. Sci. Ist. Bologna Cl. Sci. Fis. (10) 4 (1946–47), 103–109 (1948).

The author is concerned with extension of Betti's theorem of reciprocity in the infinitesimal theory of elasticity to the case of small motions without damping. So as to be able to use the method of the Laplace transform, he supposes all functions and necessary derivatives occurring to be bounded continuous functions at all points and times. He then derives a reciprocal theorem which can be formulated in the following way. In a possible motion 1 let F_1 and R_1 be the extraneous and surface-force vectors; S_1 and V_1 the displacement and velocity vectors; S_{10} and V_{10} the initial values of these latter; and let a subscript 2 denote

the corresponding quantities in a second possible motion. Then for a homogeneous body of density ρ the expression

$$ {}_0\int^t d\tau \, {}_{\mathfrak{B}}\int \rho F_1(t-\tau)\cdot S_2(\tau)d\mathfrak{B} + {}_0\int^t d\tau \, {}_{\mathfrak{S}}\int R_1(t-\tau)\cdot S_2(\tau)d\mathfrak{S} + {}_{\mathfrak{B}}\int (S_2\cdot V_{01} + V_2\cdot S_{01})d\mathfrak{B} $$

is invariant under interchange of the subscripts 1 and 2. The author shows that this theorem includes as special cases his former results on this subject [Ann. Mat. Pura Appl. (4) 18, 173–200 (1939)]. Among other special cases he notes also that in the case $R_1 = R_2 = 0$, if $\rho F_1 = G(t)a_1$, $S_{01} = \lambda a_1$, $V_{01} = \mu a_1$, where a_1 is a vector field, and similarly $\rho F_2 = G(t)a_2$, etc., then Betti's theorem remains valid in the dynamical case. The form to which the general theorem immediately reduces in the case of free oscillations is also quite simple. C. Truesdell, USA

843. F. Jung, The Culmann and Mohr circles (in German), Öst. Ingen.-Arch. 1, 408–410 (1947).

844. Cahit Arf, On the determination of the equilibrium states of a plane elastic medium with free boundaries under constant stress (in French, with Turkish summary), Rev. Fac. Sci. Univ. Istanbul (A) 12, 309–344 (1947).

Consider a plane elastic medium in equilibrium, body forces being absent. If $z = x + iy$, then the components σ_x, σ_y and τ of the stress tensor are known to be given by

$$ \sigma_x = \Re[2f(z) + \psi(z) - \bar{z}f'(z)], $$

$$ \sigma_y = \Re[2f(z) - \psi(z) + \bar{z}f'(z)], $$

$$ \tau = \Im[-\psi(z) + \bar{z}f'(z)], $$

where $f(z)$ and $\psi(z)$ are analytic functions. The author proposes to discuss in a series of papers the problem of determining the functions $f(z)$ and $\psi(z)$ defined on plane domains with free boundaries, i.e., such that $4\Re f(z) = \sigma_x + \sigma_y = 4\alpha$, where α is a constant. The entire discussion in this first paper is limited to the case when $f(z)$ reduces to a constant α over the whole plane, and not merely on the free boundaries. Under this assumption, the paper contains the solution of the following special problem (A): to determine the free boundaries L of plane simply connected domains C which satisfy the following conditions: (a) C does not contain ∞ and as z tends to ∞ along a continuous path lying in C, then arg z tends to one of the n distinct real numbers $\varphi_1, \ldots, \varphi_n$; (b) to each φ_j corresponds a finite set of disjoint intervals $(a_1^{(j)}, b_1^{(j)}), \ldots, (a_{\nu_j}^{(j)}, b_{\nu_j}^{(j)})$, consisting of the limiting values of $\Im e^{-i\varphi_j}z$ as z tends to infinity along a continuous path lying in C on which arg $z \to \varphi_j$; (c) L possesses at most a finite number of inflection points and cusps; (d) the tensor $(\sigma_x, \sigma_y, \tau)$, i.e., the analytic function $(4\alpha)^{-1}(\sigma_x - \sigma_y - 2i\tau)$ has a definite value (finite or infinite) at each point of $C + L - \infty$, and is infinite at, at most, a finite set of points of C. It is found that a solution of problem (A) can be given in terms of each analytic function $F(T)$ defined in the half plane $\Im T \geq 0$ whose derivative $F'(T)$ is a rational function satisfying certain conditions (which cannot be enumerated here) and that all solutions of problem (A) fall in this category.

Courtesy of Mathematical Reviews J. B. Diaz, USA

845. G. N. Polozhii, Solution of the third basic problem of the plane theory of elasticity for an infinite plane with a square opening (in Russian), Prikl. Mat. Mekh. 13, 297–306 (1949).

The third fundamental boundary-value problem of plane theory of elasticity is concerned with the determination of stresses in the interior of a two-dimensional region R, when the tangential stress is specified on the boundary C of R, and when displacements in the direction of the normal line to C are known. This problem was solved by N. I. Muschelishvili [Doklady Akad. Nauk SSSR 3 (1934)] for such regions R that can be mapped conformally on a unit circle by means of rational functions. In this paper the third boundary-value problem is solved for an infinite plane region bounded by a square. The solution is obtained by reduction to the solution of known boundary-value problems in the theory of functions of a complex variable. To ensure the existence of solution it is necessary to impose certain hypotheses concerning the rate of growth of stresses in the neighborhood of the angular points of the region. I. S. Sokolnikoff, USA

846. Luigi Sobrero, On the behavior of plane elastic systems in the neighborhood of reentrant corners (in Italian), Rend. Sem. Fac. Sci. Univ. Cagliari 17, 67–87 (1948).

The author is concerned with the behavior of the stresses near the vertex in a plane elastic system containing a corner, particularly in the case when the vertex angle is greater than 180 deg. The author finds series expansions for Airy's stress function F such that on the infinite wedge $\theta = \pm\alpha$ the conditions $F = 0$ and $\partial F/\partial\theta = 0$ are satisfied. Determination of the characteristic value involves a study of the equations $z^{-1}\sin z = \pm(2\alpha)^{-1}\sin 2\alpha$. The author determines the roots of these equations having smallest positive real part. From the results of this investigation the author shows that for a vertex angle $2\alpha < 180$ deg the stresses vanish at the corner, but that for a vertex angle $2\alpha > 180$ deg they generally become infinite. He proves further that if

$$ 270 \text{ deg} \leq 2\alpha < 360 \text{ deg}, $$

the stresses vary along a radius vector as the $-\frac{1}{2}$ power of the distance from the vertex, without sensible error. The author then calculates the behavior of the lines of constant principal normal and tangential stresses, sketching them for the case $2\alpha = 270$ deg. He states that these theoretical results are fully confirmed by photoelastic experiments. O. Truesdell, USA

847. A. Timpe, Nontorsional, axisymmetric deformation of bodies of rotation and its inversion (in German), Z. angew. Math. Mech. 28, 161–166 (June 1948).

The somewhat complicated structure of the equations dominating the theory of elasticity, due to the tensor character of the fundamental quantities (stress and strain), may be reduced with the aid of a generating function satisfying the bipotential equation, in the practically important cases of the two-dimensional problem and that of rotational symmetry. Having treated the case of torsion symmetric around an axis previously, the author now develops a corresponding analysis for torsionless deformation of rotational symmetry in the case of rotational bodies, terminating in the bipotential equation, accompanied by the possibility of inversion. Conclusion: The bipotential equation bears the same relation to tensor fields that the potential equation does to vector fields. Frederick Seitz, USA

848. V. K. Prokopov, The equilibrium of an elastic axisymmetrically loaded thick-walled cylinder (in Russian), Prikl. Mat. Mekh. 13, 135–144 (1949).

The object of the paper is to construct solutions suitable for the investigation of the equilibrium of axisymmetrically loaded thick-walled circular tubes of finite length. In case of axisymmetric deformations, the displacements and stresses are expressible in terms of one biharmonic function. The author constructs such a function in the form involving Bessel's and modified Bessel's functions of complex arguments, which corresponds to the absence of external forces on the lateral surfaces of the tube. He makes use of this solution to study the equilibrium of a moderately thick cylindrical shell with undeformable ends, subjected to an internal

pressure. The results are compared with those given by the usual theory of shells based on the Kirchhoff-Love hypothesis and are in good agreement when the tube is thin.

I. S. Sokolnikoff, USA

849. **Rosa M. Morris, Some general solutions of Saint Venant's torsion and flexure problem. III,** Proc. London math. Soc. 51, 424–439 (1949).

In parts I and II [same source, 46, 81–98 (1940); 49, 1–18 (1945)] the author considered the torsion and flexure of uniform beams the cross section of which could be mapped by

$$z = \Sigma a_n e^{(n-1)i\zeta}$$

on a strip in the $\zeta = (\xi + i\eta)$-plane. The present paper extends the results to include hollow cylinders whose section is bounded by two of the net surfaces $\eta = \alpha_1$, and $\eta = \alpha_2$. Complex potentials $\Omega_1(\zeta)$ and $\Omega_2(\zeta)$ are found in series form satisfying the required boundary conditions of the flexure problem. Applications are made to annuli bounded by two confocal ellipses, two eccentric circles, and two cardioids. The center of flexure is found in the two latter cases.

Courtesy of Mathematical Reviews D. L. Holl, USA

850. **S. G. Lekhnitskiĭ, Stress distribution in an elastic rod with curvilinear anisotropy under the influence of a tension force and bending moments** (in Russian), Akad. Nauk SSSR Prikl. Mat. Mekh. 13, 307–316 (1949).

A cylindrical rod of arbitrary cross section is made of elastic material possessing cylindrical anisotropy. The axis of anisotropy, in general, does not coincide with the axis of the cylinder, but the planes of elastic symmetry are normal to the generators of the cylinder. The lateral surface of the rod is free of external forces and the distribution of forces on the ends of the rod is statically equivalent to a single force directed along the axis of the rod, and a bending couple. A long isotropic rod subjected to such a distribution of forces experiences compression and pure bending, but in a long anisotropic rod in addition to bending and compressive stresses new stresses arise. These are typical of those that appear in the study of a plane deformation.

The author's formulation of the problem of compression and bending of rods with cylindrical anisotropy, leads him to the determination of the stress function satisfying a fourth-order partial differential equation similar to that appearing in the analysis of plane deformation in anisotropic elasticity. The problem of compression of thick circular pipes by longitudinal forces is solved explicitly. One peculiarity of the solution is that for certain values of the elastic coefficients the concentration of stress appears in the neighborhood of the axis of anisotropy. The problem of bending of thick circular pipes by terminal couples is also completely solved for one case of cylindrically orthotropic medium.

I. S. Sokolnikoff, USA

851. **Shunzo Okamoto, An elastotheoretical study of the stress distribution around a liningless tunnel, I, II** (in Japanese), J. civ. Eng. Soc. Japan, special issue 1947, 1948: 60–79, 159–174.

A mixture of sand and agar-agar is cast in a box and a straight hole drilled through this mixture. When this perforated block is compressed, a crack grows from the periphery of the hole. Putting $z = x + iy = ae^w + be^{-w} + ce^{-nw}$, where $w = \alpha + i\beta$, a,b,c being real constants and n a positive integer, the curve $\alpha = 0$ is assumed as the periphery of the hole. The author calculated the stress distribution in a two-dimensional gravitating elastic body having such a hole, performed the experiments mentioned, and discusses the development of a crack. He concludes that: (1) the crack occurs at the periphery where either the stress or stress gradient is large; (2) the crack develops in the direction of the longest diameter of the region in which the calculated stresses satisfy the experimental condition of rupture (the shearing stress is a certain linear function of the normal stress); and (3) a sudden change of the direction of crack is due to the concentration of stresses at the extremity of the crack. T. Mogami, Japan

852. **Tomio Itō, On the stress distribution at the walls of two parallel tunnels, without lining, of circular cross section** (in Japanese), Kensetu Kōgaku (Construction Engineering) no. 2, 17–26 (Jan. 1948).

The author calculates the stress distribution on the basis of two-dimensional elasticity theory using bipolar coordinates, the two tunnel walls being taken to be coordinate circles. The centers of the tunnels are considered to lie beneath the surface of the earth by ten times the radius r, hence rigorous satisfaction of the boundary conditions at the earth surface is not considered. When the distance between the centers is $4r$, the result is nearly the same as that already obtained for a single tunnel. Even in the case when the distance is $\sqrt{2}r$, the increase of maximum stress is shown to be slightly less than 20%. Sigeiti Moriguti, Japan

853. **S. S. Manson, Direct method of design and stress analysis of rotating disks with temperature gradient,** Nat. adv. Comm. Aero. tech. Note no. 1957 (1949).

A method for numerical solution of rotating-disk problems, such as appear in aircraft gas turbines. The method applies particularly to the inverse problem, where the disk profile is determined on the basis of an assumed stress distribution. The method applies directly to the elastic case but includes provision for temperature gradients, variable modulus, and variable Poisson's ratio. The plastic case is also considered, but the underlying theory for this is not fully clear.

The compatibility equation is written in finite-difference form. By selecting intervals for constant values of $(r_n - r_{n-1})/2$, it may be plotted in convenient design charts which give the relations between tangential and radial stress. Values of the octahedral shear stress are indicated on the same charts.

Having obtained a compatible set of stresses, the disk thickness is determined by a similar procedure applied to the condition of equilibrium.

The method is illustrated by a practical example.

C. Richard Soderberg, USA

Experimental Stress Analysis

(See also Revs. 875, 877, 885, 903)

854. **S. O. Tsobkallo and D. M. Vasilĭev, Residual-stress measurement by excision of small cylinders** (in Russian), Zavodskaya Lab. 15, 199–207 (Feb. 1949).

A method is described of measuring residual surface stresses in large pieces. By means of a machined circular slot a cylindrical portion is isolated from the material but remains undetached at its base. Residual stresses on the free, plane surface of the cylinder are measured by cementing electric-resistance gages to the surface before cutting, and measuring the strains produced by the cut. The authors attempt to establish the required ratio of depth of cut to diameter of cylinder such that the stresses acting on the uncut base of the cylinder will not significantly influence the measured strains. To treat the problem analytically, they introduce rather far-reaching simplifications in replacing the actual situation by a solid infinite cylinder, part of whose surface is acted upon by uniform, radial pressure. They find that at a distance from the loaded portion equal to about half the diameter,

the radial strain is about 1% of the maximum radial strain in the loaded portion, a result which is hardly surprising in view of Saint Venant's principle. They conclude, by implicit analogy, that a depth of cut equal to the diameter is more than sufficient to insure undisturbed surface strains. A test is described to verify this contention. However, in the test two long straight slots were used to isolate the test portion, rather than the cylindrical slot for which the analysis was made. Difficulties are described of obtaining reliable strain measurements over a sufficient length of time (zero-shift); this error, for 100 hr, amounted to 3% when bakelite cement heated to 70 C for 20 hr was used, and to 13% for both bakelite and celluloid-acetone cements dried at room temperatures. It is maintained that if X-ray stress measurements were made instead, a cylinder diameter of 1 mm would suffice, but problems of accuracy of this method are not discussed.

George Winter, USA

855. Eric Jones, Some physical characteristics of the wire-resistance strain gauge, Proc. 1946 Conf. Manchester and District Branch Inst. Phys. Lond., 1–26 (1948).

This paper gives a clear and rather complete (though concise) investigation of the various common types of wire-resistance strain gages. Their advantages, properties and uses are described, suggestions as to their most efficient use are given, and their behavior under various conditions is investigated from both a practical and a theoretical viewpoint.

After a brief description of the gage itself, a separate analysis is given of the behavior of the three essential elements of the gage: the resistance wire, the matrix in which the wire is embedded, and the adhesive layer between the gage proper and the specimen.

The investigation of the wire leads to values for the gage factor, which is found to depend on the change in the dimensions of the wire due to strain, on the variation of wire resistivity with strain, on the geometrical arrangement of the gage winding, and on the orientation of the gage axis relative to the principal axes of strain in the specimen.

The analysis of the behavior of the matrix and of the adhesive layer includes investigations of the stress distribution in the gage, of imperfect gluing in various locations, of changes in humidity and temperature. The current-carrying capacity and the optimum resistance are discussed, as well as the limits of frequency response and of linearity. A brief treatment of gage behavior and fatigue under repeated loadings is included. Comments on creep, drift, setting time and hysteresis, and suggestions concerning drying and waterproofing are also given. B. A. Boley, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 836, 847, 849, 850, 862, 875, 876, 878, 882)

856. J. Rieve, The distribution of stresses between the flange and the web of I-shaped beams under a concentrated load (in German), Z. angew. Math. Mech. 28, 210–217 (July-Aug. 1948).

Expressions are developed for the normal and shear stress components along the plane of contact of the flange and web of an I-beam when a concentrated load acting parallel to the plane of the web is applied at a point in the flange. General equations for stress and strain are obtained by expressing the Airy stress function in terms of two potential functions which satisfy both the compatibility and the Cauchy-Riemann equations. The general equations are made applicable to the problem in hand by considering the conditions of stress and displacement along the plane of contact of flange and web on one half the I-section, the other half being discarded on the basis that, by Saint Venant's principle, it will have a negligible effect on the stresses under investigation.

Since the effect of the concentrated load is actually distributed over a finite length of beam, the load acting on the web is expressed in Fourier-series form, and the potential functions associated with the internal stresses are then related to it. The expressions obtained contain, among other factors, the thickness, area, and moment of inertia of the flange, the web thickness, and the period of the Fourier series. These factors are manipulated to permit the development of approximate formulas and these are shown, by an illustrative example, to be in close accord with the exact solutions. The results are applicable to the analysis of sections having a continuous connection between flange and web (extruded or welded sections) or to those having riveted connections.

Joseph S. Newell, USA

857. D. N. Mitra, Torsion and flexure of an isotropic elastic cylinder whose cross section is a semi-cardioid, Bull. Calcutta Math. Soc. 41, 28–40 (1949).

This paper provides a complete solution of the torsion and flexure problem of a prism whose cross section is a semicardioid. It employs the methods of S. Ghosh [Revs. 1: **1312, 1317;** 2: **19, 987**]. Explicit results are given for the torsional rigidity and for the coordinates of the center of flexure.

D. L. Holl, USA

Plates, Disks, Shells, Membranes

(See also Revs. 848, 864, 865, 866, 867, 868)

858. K. Goriupp, The rectangular plate supported on three sides. II (in German), Ingen.-Arch. 16, 153–163 (1948).

In continuation of his earlier work (see Rev. 2, **27**) the author now considers problems for which two or three of the previously simply supported edges are clamped. An explicit solution is now no longer possible, and the author makes use of the procedure previously used by H. Hencky in his well-known dissertation for the rectangular plate with all four edges clamped.

E. Reissner, USA

859. Piero Pozzati, On rectangular elastically-supported plates (in Italian), Boll. Un. mat. ital. 3, 236–248 (Dec. 1948).

The paper contains a Fourier-series solution of the problem of a rectangular plate simply supported all around the boundary, and on a number of evenly spaced beams parallel to its sides, loaded in an arbitrary manner. The statically indeterminate beam reactions are considered as line loads and are expanded into Fourier series, whose coefficients are obtained by equating the deflections of the plate to the deflections of the beams.

The numerical solution of a square plate, uniformly loaded and supported by four beams, indicates the rapidity of convergence of the series: the error in the maximum bending moment made by taking into account the first term only if its expansion is 8%. In a subsequent paper the author will give coefficients for bending moments and shears for plates supported on a grid of up to 5-by-5 beams.

Mario Salvadori, USA

860. Robert Sauer, Geometric remarks on the membrane theory of shells of negative curvature (in German, with Russian summary), Z. angew. Math. Mech. 28, 198–204 (1948).

It is shown that the characteristics of the differential equations of equilibrium of the linear membrane theory (momentless theory) of thin shells are the asymptotic lines of the shell surface. This confirms a result of W. Flügge [same source, 25/27, 65–70 (1947); Rev. 2, **848**] regarding the reality of the characteristics for

shells with negative curvature, in contrast to what is the case for shells with positive curvature. On the basis of this result there is outlined a procedure for the graphical determination of the state of stress in shells with negative curvature, in the absence of surface loads. Use is made of results by W. Blaschke [Proc. Fifth Intern. Congress Math. Cambridge 1912, 2, 291–294 (1913)] concerning graphical statics of membranes. Explicit solutions are obtained for the stresses in shell surfaces of the second degree, without surface loads. E. Reissner, USA

Buckling Problems

(See also Rev. 878)

861. Kōsiro Ōi, On buckling by impact (in Japanese), Ōyō Sūgaku (Appl. Math.) 2, no. 1, 13–14 (Feb. 1949).

If a long column is subjected to longitudinal impact load, it buckles in the shape of higher harmonic vibration. A simple mechanological explanation of this phenomenon is presented here. A long column with supported ends is considered. Its lateral deflection is expressed as $y = \Sigma a_n(t) \sin n\pi x/l$. Assuming the longitudinal impact load P to be constant, it is shown that the Fourier coefficient $a_n(t)$ increases as $\exp(\alpha_n t)$ where

$$\alpha_n^2 = (n^2\pi^2/\rho l^2)(P - n^2\pi^2 EI/l^2),$$

for n such that α_n^2 is positive. (For larger n, $a_n(t)$ merely oscillates.) α_n^2 is maximum for n nearest to $Pl/2\pi(EI)^{1/2}$, whence it is deduced that the most possible shape of buckling corresponds to the critical load nearest to the half of the impact load.

Sigeiti Moriguti, Japan

862. M. A. Lazard, Report of tests on buckling of bars elastically supported (in French), Inst. tech. Bâtim. Trav. publ. no. 88, 27 pp. (1949).

The results of buckling tests on rods with intermediate elastic supports are analyzed in connection with other recent buckling tests and especially with the theories developed by Engesser and Timoshenko for the compression of rods with continuous elastic supports.

The author's tests take into account the transverse elastic reactions, as well as the stiffness in torsion of the supports. The results represented in diagrams showed that Engesser's solution was straddled on both sides by these diagrams, while that of Timoshenko generally showed greater values. By means of a generalization of the notion of transverse rigidity, the author explains some particularities of the recent American tests, and shows the concordance between these last results and his own.

Aurel A. Beleş, Rumania

863. Masao Yoshiki, A new method of determining the critical buckling points of rectangular plates in compression (in Japanese), J. Soc. appl. Mech. Japan 1, 193–199 (Nov. 1948).

Experimental determination of the critical buckling load of thin plates in compression is difficult because the apparent compression and lateral deflection change rather slowly even in the neighborhood of the critical point. Southwell, Cox and Donnell have proposed their respective methods to overcome the difficulty. The author proposes here a new method, which proves to be more satisfactory than those mentioned above.

The method consists of plotting the load against the square of lateral deflection, fitting a straight line to the region after buckling, and determining the critical load as the point of intersection of the fitted line and the ordinate axis. It is based upon the analysis of the behavior after buckling of plates with small initial deflection. It is applied to rectangular plates in compression freely supported at four edges. Forty-seven plates were thus tested. Critical loads of 40 plates are determined perfectly, while those of 7 plates are somewhat ambiguous. On the other hand, by Cox's method the critical loads of 16 plates cannot be determined at all and other 10 plates are rather ambiguous. By Donnell's method, 2 plates are impossible and 18 are ambiguous.

Sigeiti Moriguti, Japan

864. W. S. Hemp and K. H. Griffin, The buckling in compression of panels with square top-hat section stringers, Coll. Aero. Cranfield Rep. no. 29, 57 pp. (1949).

Theoretical buckling curves are computed for the title problem by standard plate theory using the customary assumptions that end effects and attachment flanges can be ignored, and that the corners of the stringer remain straight. Comparisons made with British and NACA test data indicate fair over-all agreement, but the scatter of the test data and some rather wide discrepancies (50%) prevent drawing definite conclusions as to the validity of the results. Among the possible reasons for these discrepancies are the neglected end effects, the incorrect use of the tangent modulus for plastic buckling, and conceivably buckling in some mode other than that contemplated. S. B. Batdorf, USA

865. Ch. Dubas, Contribution to the study of buckling of stiffened metal sheets (in French), Mitt. Inst. Baustat. Eidgenöss. tech. Hoch. no. 23, 152 pp. (1948).

This monograph deals with the analysis of problems of bending and buckling of flat plates, both uniform and reinforced, by a special technique of approximate calculation. The essentials of this technique may be described as follows.

For the case of a uniform plate, referred to axes Oxy, bent by transverse pressure P, one considers first of all a division of the plate into distinct x-wise beams of small width Δy. These beams are assumed to have rigidity $D\Delta y$, where D is the flexural rigidity of the plate, and to be loaded by a pressure P_x. The bending moment M_x per unit width and the deflection w_x can be calculated from P_x by well-known formulas of the theory of beams. Dividing the plate into y-wise beams, one introduces similarly quantities P_y, M_y, w_y. Further let P_{xy} be the pressure applied to the x-wise beams so as to produce in them a bending moment distribution M_y, while P_{yx} is the pressure required to produce a bending moment M_x in the y-wise beams. Imposing the condition

$$w_x = w_y = w$$

(say), it is easily shown that,

$$P_x + 2P_{xy} + P_y = D\nabla^4 w = P, P_{xy} = P_{yx}$$

if w be taken as the transverse deflection of the plate under pressure P. The problem of plate bending is thus the problem of splitting P into 3 parts P_x, P_y, $2P_{xy}$, such that the deflection produced by P_x acting upon the x-wise beams is equal to the deflection produced by P_y upon the y-wise beams, while the corresponding bending moments in either set of beams are such that they are equal to the bending moments obtained by applying a pressure $P_{xy} = P_{yx}$ to the other set. This second problem is resolved by approximate numerical calculations using only the ideas and formulas of beam theory.

The writer develops the technique to deal with the case of a reinforcing member (beam) and to handle problems of buckling. The bulk of the monograph is concerned with this last type of problem. W. S. Hemp, England

866. Manuel Stein and David J. Yaeger, Critical shear stress of a curved rectangular panel with a central stiffener, Nat. adv. Comm. Aero. tech. Note no. 1972, 19 pp. (1949).

Theoretical coefficients for the critical shear stress of a simply-supported curved rectangular plate with a central stiffener which has no torsional stiffness are obtained by the Galerkin method. The stiffener is considered to be concentrated along the center line of the plate (either along the axial or the circumferential direction). Results are presented in the form of tables and curves for various curvature parameters and stiffness parameters. Comparison with unstiffened plates is discussed. C. C. Wan, USA

867. Samuel Levy and Wilhelmina D. Kroll, Primary instability of open-section stringers attached to sheet, J. aero. Sci. 15, 581–591 (Oct. 1948).

The theoretical analysis of the compressive buckling and post-buckling behavior, including failure, of plate assemblies such as stiffened panels, is at present considered impracticable and approximate methods are resorted to. If most of the load is carried by the stiffeners, it is reasonable for purposes of analysis to regard the panel as a set of stiffeners stabilized by the sheet against warping and twisting, and augmented in load-carrying capacity by some effective width of attached sheet.

This idealization is employed in the present paper. The panels are assumed to fail by torsional instability of the stiffeners, which are constrained to rotate about an axis in the plane of the sheet. An effective torsional stiffness of the attached sheet is also considered. Small-deflection theory is used so that only initial buckling can be determined, and any additional strength developed during buckling is neglected. At high stresses a correction for the plasticity of the material is made.

The method is applied to 40 panels for which experimental data are available. Best agreement is obtained when the torsional restraint of the sheet is neglected. On this basis, the maximum strength of those panels with stringer area exceeding the sheet area is predicted with an error less than about 8%. The errors for panels with sheet area exceeding stringer area are much larger, which suggests that the idealization used is not suitable for such panels. S. B. Batdorf, USA

868. S. B. Batdorf, Murry Schildcrout, and Manuel Stein, Critical stress of thin-walled cylinders in axial compression, Nat. adv. Comm. Aero. Rep. no. 887, 8 pp. (1947).

This paper is chiefly concerned with the effect of cylinder length, which becomes important when the length is less than the radius. The theoretical buckling stress is derived for perfect cylinders having hinged and fixed edges at the ends (considering the conditions for the radial displacements, but ignoring those for the other displacements). This results in a relation between the parameters $\sigma_x L^2 t/D$ and $Z = L^2/(\sqrt{(1-\mu^2)}rt)$, where σ_x, L, t, r, D and μ are buckling stress, length, thickness, radius, flexural stiffness and Poisson's ratio. For $Z > 10$ this relation coincides with the classical results obtained without considering the effect of length or end conditions.

Empirical curves, which the authors recommend for design purposes, are then found by plotting available test data against the above parameters, drawing curves for various values of r/t and fairing these into the theoretical curve for very small values of Z. It is claimed that such plotting reduces scatter, but no evidence is given that for the longer cylinders it results in less scatter than the usual practice of plotting ratio of experimental to theoretical stress directly against r/t. L. H. Donnell, USA

869. N. J. Hoff, Bruno A. Boley, and Joseph J. Mele, Stresses in and general instability of monocoque cylinders with cutouts, VII—Experimental investigation of cylinders having either long bottom cutouts or series of side cutouts, Nat. adv. Comm. Aero. tech. Note no. 1962, 57 pp. (1949).

The report presents experimental data on the nonlocalized type of buckling (general instability) of airplane fuselages with large cutouts. Eight 24S-T alclad cylinders 20 in. in diameter, with 0.012-in. sheet and numerous stringers and rings, were tested to destruction in pure bending. Some of the cylinders had long cutouts on their compression sides, and others had rows of shorter cutouts on both sides. These simulated bomb bays and windows in fuselages. In all cases, except one, the rings participated in the buckling. In addition to the ultimate loads, strains in the stringers at several stations were recorded. These data were obtained with SR-4 electric strain gages.

H. L. Langhaar, USA

870. N. J. Hoff, Bruno A. Boley, and Merven W. Mandel, Stresses in and general instability of monocoque cylinders with cutouts, VIII—Calculation of the buckling load of cylinders with long symmetric cutout subjected to pure bending, Nat. adv. Comm. Aero. tech. Note no. 1963, 36 pp. (1949).

The report treats buckling, under pure bending, of a long cylindrical shell, reinforced with stringers and rings, and having a long symmetric cutout on the compression side. Attention is restricted to modes of buckling in which the rings and the stringers participate. A strain-energy analysis of the problem is developed. The wave length of a bulge is employed as one of the parameters whose magnitude is determined by the condition that the buckling load is a minimum.

Many known discrepancies between theoretical and experimental results in the buckling of curved shells have led some investigators to the belief that the elementary concepts of elastic stability theory are not a practical basis for the treatment of this type of problem. Possibly the authors' results reflect this circumstance. In four tests, the calculated buckling loads exceeded the observed buckling loads by amounts ranging from 18% to 45%.

H. L. Langhaar, USA

871. G. Wästlund and S. Bergman, Buckling of webs in high steel girders (in Swedish), Tekn. Tidskr. 77, no. 7, 869–874 (Nov. 1947).

The authors point out that all the webs of a steel girder are subjected mainly to boundary forces acting in the plane of the webs, but that the distribution of these forces is vastly different for the various webs, according to their position relative to the end and the intermediate supports of the bridge-girder. Three distinct, typical cases of load distribution were studied. Not only photographs but also rather accurate measurements of the buckled surfaces were made and the buckling loads studied. In addition the "practical rupture loads," that is the loads under which considerable permanent deformations make their appearance, were determined. New rules for the design of such bridges are suggested by the authors. P. F. Neményi, USA

872. Kenneth P. Buchert, Stability of Alclad plates, Nat. adv. Comm. Aero. tech. Note no. 1986, 33 pp. (1949).

Sheets of high-strength aluminum alloy are commonly covered with a considerable thickness of nearly pure aluminum as a means of providing better resistance to corrosion. The buckling of alclad plates formed in this way is measurably affected by the fact that the pure aluminum located at the outer fibers becomes plastic at a low stress.

In this technical note the theory of buckling of homogeneous sheets is extended to the case of a plate with layers of materials with different stress-strain curves, and the results cover both the elastic and the plastic state. The available experimental results for alclad plates are then compared with the theory developed, and good agreement is found.

A discussion is given of the stress-strain relations for the two materials, as needed in making calculations for alclad plates, and a simple formula is given for determining the percentage of outside covering from a known composite stress-strain curve. A good deal of calculating work remains to be done before the analysis and formulas given in this report are in a form to be readily applied by the practicing stress analyst.

Wm. C. Johnson, Jr., USA

Joints and Joining Methods

873. P. L. J. Leder, Factors influencing the weldability of high tensile alloy steels, and a new weld cracking test, Proc. Instn. mech. Engrs. 159, no. 40, 173–190 (1948).

This paper is concerned with a single phase of the weldability question, viz., cracking due to restraint. A weld-cracking test is described which contains a butt weld under transverse restraint. According to the author, the test embodies two important new features: It measures the stress across the weld directly, and it takes into account the fact that when a pair of plates are butt-welded, a couple at right angles to their plane is superimposed on the main stress.

By means of this test it was shown that: (1) in the absence of cracking, the load-versus-time curve for a given size of weld was of essentially constant shape and dimensions; (2) in the case of a cracked weld, the curve initially followed the same path as in an uncracked weld; and (3) the stress levels attained in an uncracked weld were sometimes higher than in a cracked weld. From the latter, it was concluded that stress level alone was not responsible for cracking in the hardened zone. [For an appreciation of the highly localized stresses that exist at a weld, see Nippes and Savage, Weld. Res. Suppl. 13, 370–376 (1948); Rev. 1, 1116]. A study of the effect of hydrogen confirmed the findings of other investigators, viz., that hydrogen can be a most important factor in hardened-zone cracking. The author points out, however, that cases arise where steels of similar composition exhibit marked differences in their tendency to crack when hydrogen is held essentially constant.

Carl E. Hartbower, USA

874. Stefan Soretz, Electrical flash welding of cold-worked and nonhardened steel for reinforcing concrete (in German), Z. Öst. Ingen.-Archit.-Ver. 94, 6–13 (Jan. 1949).

In this paper the author gives the results of static tests of flash-welded mild steel for reinforcing concrete of circular cross section with two ribs (Torstahl) of ultimate strength 52,8-55,2 Kg/mm². On the basis of his own experiments and those of Hauttmann he arrives at the following conclusions: (1) Manually operated flash welders give welds the strength of which is 20% below the ultimate strength of the unwelded material. For fully automatic flash welders the corresponding figure is 10%. (2) It is possible to restore the full strength of manually flash-welded parts by means of cold work. The best procedure to achieve this is to remove the seam and to twist a 6-in. length containing the weld through an angle of 30 deg. This also affords a test of the quality of the weld: if cracks appear the weld is rejected.

M. M. Gololobov, Czechoslovakia

Structures

(See also Revs. 859, 866, 869, 870, 871, 874, 896)

875. Gustavo Colonnetti, Theory of structures (Scienza delle costruzioni), 2nd ed., Rome, Giulio Einaudi, 1948, 562 pp., 137 figs. Paper, 6.3 × 9.8 in.

This book is concerned mostly with presentation and discussion of fundamental principles regarding elastic and elastoplastic media. The only "structures" specifically treated are beams and arches. All theoretical principles are presented as general theorems in mechanics of continua. The author's aim is to give the reader sufficient understanding of the basic principles so that he can make his own applications to engineering structures. Thus the various theorems are not specialized to frames, trusses, etc., and there is no mention of numerical methods of solution, of buckling problems, and various other practical matters.

The first half of the book presents the foundations of classical theory of elasticity. Prominence is given to properties of the "elastic potential energy" (strain energy) defined merely as a positive quadratic form in either the six strain components or the six stress components. Thus "Menabrea's theorem" (of least work) and Castigliano's theorem are developed before Hooke's law is mentioned and the isotropic moduli are introduced. There follows a very complete and detailed presentation of the classical solutions of Saint Venant's problem, and then two chapters on technical theory of beams. The third quarter of the book is devoted to analysis of beams and arches approached by way of two reciprocal theorems and the "ellipse of displacements."

The last quarter of the book concerns elastoplastic phenomena. Strong emphasis is put on a minimum principle due to the author, according to which a certain function of the stresses (satisfying equilibrium with the external loads) is rendered a minimum by the actual stresses. In this function the plastic strains appear explicitly, and are treated as constants. The author applies this theorem to determine reactions of beams and arches considering plastic strains and loads as known. Since he characterizes this principle as the central theorem in the theory of elastoplastic equilibrium, it is regrettable that he does not explain how it is to be used in any problem in which the plastic strains are not known. The author discusses briefly the possibility of taking advantage of plastic deformation in a statically indeterminate structure to obtain more economical designs. He does not mention the work done in roughly the past 15 years (Baker, Maier-Leibnitz, van den Broek) which has brought these ideas to very practical realization.

P. S. Symonds, USA

876. W. Swida, A method to determine the deformations of structures in the elastoplastic state (in German), Z. angew. Math. Mech. 27, 168–169 (Aug.-Sept. 1947).

The paper presents "area-moment" and energy formulas for partly plastic beams, and formula for the stresses in partly plastic curved beams under pure bending. Derivations and explanations of symbols are very sketchy.

Ed.

877. E. Schmidt, Model tests to determine the dimensions of structures, Schweiz. Bauztg. 67, no. 39, 555–561 (1949).

It is not always possible nor practicable to calculate the strength and stiffness of all building structures. In many cases model tests may be performed to check calculations, to save time, etc. Model materials include steel, light alloys, celluloid and plaster of Paris. The adaptability of these materials is discussed. The deformations are measured by the aid of the deformeters of Beggs and dial-gage indicators, the curvatures with a special constructed curvimeter, and the slopes by the aid of an optical slope indicator. A number of applications with various models is discussed. The results are in good agreement with the theoretical values.

R. G. Boiten, Holland

878. J. F. Baker, A review of recent investigations into the behavior of steel frames in the plastic range, J. Instn. civ. Engrs. 31, 188–240 (Jan. 1949).

The paper gives a summary of the results of recent studies, con-

ducted primarily in Great Britain, of what is known in this country as "limit design." The well-known procedures for continuous beams are extended to the design of frames subjected to vertical and horizontal loads. Safety criteria, wind intensities, etc., are discussed and the results of experiments on full-scale models are given. The paper does not discuss the question of residual stresses or fatigue. E. F. Masur, USA

879. J. J. Thompson and W. H. Wittrick, The stresses in certain cylindrical swept tubes under torsion and bending, Counc. sci. indust. Res. Aero Res. Rep. no. ACA-43, 20 pp. (1949).

The advent of swept wings on high-speed airplanes in recent years has introduced new and rather difficult problems in stress analysis for the aeronautical engineer. A comprehensive treatment of the secondary stresses in straight thin-walled beams has been given by Hadji-Argyris and Dunne [J. Roy. Aero. Soc. 51, p. 199 (1947); Rev. 1, **626**]. This theory was extended to swept beams by Wittrick [same source, ACA-39 (1948); Rev. 3, **54**]. The equations that must be solved for arbitrary loading and arbitrary cross sections were found to be very complicated. In the present paper two cases are considered in which the loading and cross sections are very simple.

The swept beam is assumed to have internal rigid diaphragms which are skewed with respect to the axis of the beam. This corresponds to a swept wing with ribs that are parallel to the plane of symmetry of the airplane. The Saint Venant problems of pure bending and pure torsion are well known for the straight wing. The corresponding problem for the swept wing is the first problem treated in this paper. The applied end torques and end moments are defined with respect to a plane which is parallel to the ribs. A trapezoidal thin-walled cross section with single symmetry is considered. Axial stresses are carried by the walls and also by four corner flanges. The axial displacements, normal stresses and shear flows vary linearly along the walls. It is surprising to find axial stresses due to torque, and also uniformly distributed shear flows due to the bending moments, in planes parallel to the ribs. This problem provides a basic introduction to the stress analysis of swept beams, and thus is of historical interest.

The second part of the paper treats a beam with a rectangular cross section having double symmetry. A solution is given for an exponential spanwise variation of section torque. Since the walls, as well as the flanges, are assumed to carry normal stresses, the solution requires extensive mathematical effort. The mathematical difficulties which arise with very simple cross sections show clearly that much research is needed to develop practical methods for the stress analysis of swept wings.

Stanley U. Benscoter, USA

880. Fernand Campus, A general theory of plane elastic arches (in French), Hommage Faculty appl. Sci. Univ. Liége, 195–220 (1947).

Suppose x, y and ω to be the coordinates of an arbitrary point on a plane arch, and α, β and φ the coordinates of a special point. A and B are the extremities of the arch. According to Bresse, the following equations hold true:

$$\Delta\alpha = \Delta x_A - (\beta - y_A)\Delta\varphi_A - \int_A^\alpha \frac{M}{EI}(\beta - y)ds - \int_A^\alpha \frac{N}{E\Omega}\cos\omega\, ds + \int_A^\alpha \frac{T}{G\Omega'}\sin\omega\, ds,$$

$$\Delta\beta = \Delta y_A + (\alpha - x_A)\Delta\varphi_A + \int_A^\beta \frac{M}{EI}(\alpha - x)ds - \int_A^\beta \frac{N}{E\Omega}\sin\omega\, ds + - \int_A^\beta \frac{T}{G\Omega'}\cos\omega\, ds,$$

$$\Delta\varphi = \Delta\varphi_A + \int_A^\alpha \frac{M}{EI}\, ds.$$

From these equations the author derives a very general theory for the calculation of plane arches with end conditions varying from hinged to built-in. The effects of uniform or nonuniform temperature variations are also included. The method is illustrated with a number of classic examples, symmetric parabolic arches with constant reduced height, parabolic arches with constant reduced section and moment of inertia, circular arches with constant thickness and breadth, etc. It is shown that arches with two different end conditions, one hinged and one built-in, may be treated in the same way. Furthermore, a solution is presented for continuous arches with fixed or moving points of support.

R. G. Boiten, Holland

881. Albert de Marneffe, Method of calculation of the skewed statically indeterminate arch (in French), Hommage Faculty appl. Sci. Univ. Liége, 221–227 (1947).

This analysis of the skewed statically indeterminate arch assumes that the two principal moments of inertia of the cross sections are equal and that the twisting is governed by the polar moment of inertia. Its application is thus practically restricted to three-dimensional pipe bends. The author uses throughout the geometric language of elastic masses and ellipses of elasticity. Neglecting the elastic displacements due to normal and shearing forces, he proves the existence of an ellipsoid representing the rotational deformability of the arch.

The further study is strongly simplified by taking the principal axes of this ellipsoid as coordinate axes, as by the use of the elastic center in the plane case. The general expressions for the displacements of a point are then written down, taking separately into account the effect of exterior forces and of the three statically indeterminate reactions; these equations generalize the well-known Navier-Bresse equations of the plane arch. By writing down the end conditions, one has finally to solve a system of three linear equations.

The method is applied to a built-in three-dimensional pipe bend, composed of three orthogonal branches. The case of an uniform thermal dilatation is followed by the study of the action of an external force directed along the central branch of the pipe.

Charles Massonnet, Belgium

882. S. Erlik, Semi-circular bow girders, Concr. Constr. Engng. no. 6, 64, 169–174 (June 1949).

Solutions are given for a concentrated load at any point, or a uniformly distributed load, perpendicular to the plane of a uniform 180-deg bow girder having fixed ends. Design charts for the resulting bending and twisting moments are shown. Ed.

883. R. Gran Olsson, Remarks on the deflection theory of suspension bridges, Reissner Anniv. Vol., J. W. Edwards, Ann Arbor, 211–230 (1949).

The author has developed a method for integrating Rode's equation,

$$(EJ\eta'')'' - (H_w + H)(1 + y'^2)\eta' = Hy'' + p(x),$$

for suspension bridges, which takes into account the horizontal movement of the cables, neglected in the classical equation of J. Melan.

He reduces it to a second-order differential equation in terms of η'. Then, assuming the cable to be parabolic in shape, and the EJ of the truss to be constant, a solution of Rode's equation is given in terms of tabulated functions.

A solution can be obtained in some cases for a variable EJ of the truss where $EJ = EJ_0(1 + y'^2)^n$ (n an integer). For n equal to -1, $+1$, $+3$ the solutions are given by elementary functions, for $n = 2$ by Legendre polynomials with purely imaginary argument, for $n = 4$ by the spheroidal wave functions, and $n = 0$ by Whittaker functions or Hermitean polynomials. Rode's equation may also be solved for a stiffness varying according to

$$EJ = EJ_0(\xi/\xi_0)^n$$

where J_0 corresponds to ξ. For $n = 2$ and $n = 4$ the solution is given by cylinder functions with a purely imaginary argument.

The author states further, that a solution to Rode's equation can be obtained for a constant and, in some cases, for varying stiffness of the truss when the cable is assumed to be a catenary.

It has been pointed out that the eigenvalues and eigenfunctions of the differential equation of suspension bridges are identical with the eigenvalues and eigenfunctions of the differential equation of the stiffening girder when subject to axial buckling as a slender column. In previous works only the differential equation of Melan has been considered. The author has extended the analogy to cover Rode's equation, by introducing a friction force increasing linearly from the middle of the column towards both ends and acting parallel to the end compression load. The compressive forces in the analogous column will vary according to a parabola corresponding to the parabolic form of the cable.

Karl Arnstein, USA

884. **H. W. Swift, Elastic deformation of piston rings,** Engineering Lond. 163, 161–165 (1947).

The deflection characteristic of a piston ring is discussed. It is pointed out that any method of wall-pressure measurement which permits the least displacement from the true surface of the cylinder is very inaccurate. A graphical method to determine the correct geometric form of the ring is also presented.

C. C. Wan, USA

885. **Antonio Signorini, On noncylindrical dams** (in Italian), Pubbl. Ist. Appl. Calcolo no. 185 = Pont. Acad. Sci. Comment. 10, 375–406 (1947).

After a discussion of the practical obstacles to a mathematical study of the stability of reinforced-concrete dams, the author accepts the mathematization of F. Tölke [*Talsperren und Staumauern*, Springer, Berlin, 1938]. He argues, however, that Tölke's reduction to a boundary problem for a single fourth-order partial differential equation cannot be regarded as satisfactory. He shows that the problem should be reduced to a system of two linear equations, one fourth-order partial and one third-order ordinary, if certain reasonable geometric restrictions are assumed.

Courtesy of Mathematical Reviews B. Levi, Argentine

886. **F. Campus and R. Jacquemin, Fatigue tests on railway sleepers of reinforced concrete or prestressed concrete** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 2, 133–145 (1947).

Laboratory experiments with railroad sleepers of reinforced and prestressed concrete are described. Primary objective was the determination of the endurance limit of these sleepers. The sleepers consisted of three parts, two massive concrete blocks under the rails and one slender concrete bar with semiflexible end joints connecting the two blocks. The end joints were formed by crossed reinforced-steel bars. This construction permits an almost independent deflection of both rails. Basically two types of testing machines have been used to obtain endurance limit values. One machine tested primarily the two blocks under the rails, and a second machine tested the two end joints of the connecting bar. The first machine subjected the sleepers in a horizontal position to vertical loads by means of an Amsler pulsator. The loads varied between a minimum of 1 ton and a maximum of 25 tons at 500 cpm. The second machine investigated the sleepers in a vertical position, clamped rigidly at the upper end. A horizontal load was induced by rotating weights eccentrically supported. The axis of rotation of these weights was attached to the lower block of the sleepers and produced deflections up to 20 mm. Most tests were performed at half the resonance frequency of the vibrating system. The first test series showed that the experimental sleepers did not show any fatigue failure after 6×10^6 pulsations. The second test series indicated that the joints endured 25×10^6 pulsations of 15 mm displacement amplitude at 270 cpm without rupture. R. K. Bernhard, USA

887. **L. Fox, Computation of traffic stresses in a simple road structure,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 236–246 (1948).

The problem of stress analysis in a two-layer slab system, such as a concrete pavement of finite thickness supported on a uniform soil layer of infinite thickness, is investigated by an exact and an approximate numerical method. The solutions obtained are for a circular uniform loading away from an edge or corner where radial symmetry exist. Of special interest is the comparison of the results for a frictionless and a perfectly rough interface (bottom of slab). Useful graphs are included. The exact solution gives results only on a vertical axis below the center of the loading. Other values were found by numerical analysis. L. E. Grinter, USA

888. **K. W. Mautner, Structures in areas subjected to mining subsidence,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 167–177 (1948).

Settlements of foundations due to mining operations are discussed in this paper. The factors which influence such settlements are examined and the effect of such settlement on structural design is indicated. Examples and performance records of structures located in areas where subsidence due to mining operations has occurred are presented. Ralph E. Fadum, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 858, 866, 875, 897, 898, 899, 900)

889. **Albert Kammerer and André Lamare, The deviations from Hooke's law and the limits of the reversibility field** (in French), C. R. Acad. Sci. Paris 229, no. 22, 1121–1122 (1949).

In a previous note [same source, 227, p. 1144 (1948); Rev. 2, **467**] the first of the authors proposed to modify the elastic equations by introducing a coefficient accounting for the viscous effects and assuming that the elastic modulus depends on the strain. The rupture load should then depend on the loading rate, as is observed with plexiglas. In a bending test, the neutral line should be displaced, and the maximum tension reduced in the ratio $v(\Omega/I)^{1/2}$, relative to the classical results (here v is the distance of the center of gravity from the extreme fiber, Ω the cross-sectional area, and I the moment of inertia). The proof of this

statement is not given, but it is confirmed by experiments on samples of various shapes. The theory seems to hold much promise.

Jean Goguel, France

890. F. Vitovec, Strain hardening and residual stresses of the third kind (in German), Öst. Ingen.-Arch. 3, no. 2, 119–128 (1949).

The author reviews in this paper the various theories which were proposed by physicists to explain the ductility of metallic single crystals and crystal aggregates. He describes the various physical models of crystals in which certain irregularities were assumed in the lattices of the atoms, such as G. I. Taylor's dislocations, the models containing flaws or cracks (Griffith, Smekal) and Zwicky's "mosaic" structure of blocks of atoms. He distinguishes these real crystals, having all kinds of imperfections in their lattices, from the ideal crystal. He attempts to utilize these hypotheses for explaining a plausible mechanism for the phenomenon of the strain hardening of the ductile metals. He distinguishes three types of residual stresses in plastically deformed metals: the residual stresses of the first kind, acting in microscopic regions; those of the second kind in microscopic regions; and those of the third kind which act in zones of the order of magnitude of a few lattice distances of the atoms. To the development of this third type of stresses during a plastic distortion he attributes the main causes of the phenomenon of the strain hardening of the metals.

A. Nadai, USA

891. A. Kochendörfer, On the theory of slide hardening (in German), Z. Phys. 126, 548–568 (1949).

The theory of strain hardening presented by the author in 1937–1938 and applied in this country, for example, by Nowick and Machlin (Nat. adv. Comm. Aero. Rep. no. 845, 1946) has been further developed by the author to account for more of the phenomena associated with plasticity and high-temperature creep.

In this article the revised theory is presented, and also a model to illustrate the main features of the theory, which may be summarized as ascribing the hardening to the action of stuck dislocations in opposing the generation and movement of other dislocations. The model, complete with springs and cranks so arranged as to give proper phase relations, is described in detail and the strengths of the springs are evaluated in terms of the deformation energies of the active and inactive atoms around the dislocation. The revised theory seems to be in harmony with a wide range of experimental data, although the comparison of results is as yet largely qualitative.

Finally, the author compares his theory with that of Mott and Nabarro (Rep. Bristol Conf. Phys. Soc. London, 1948) and remarks that these two theories tend to establish two different kinds of hardening: one caused by an inner stress field, the other caused by interaction between the dislocations.

Wm. C. Johnson, Jr., USA

892. G. Kolesnikov, V. Pavlov, E. Yakovleva and M. Yakutovich, Plastic deformation and disintegration of polycrystallic metals at elongation (in Russian), J. tech. Phys. 19, no. 1, 62–75 (Jan. 1949).

This paper describes an investigation into the influence of temperature and speed of deformation on the shape of the stress-strain diagram. Only pure polycrystallic metals were tested, namely, β-Sn, Cd, Zn, Mg, Pb, Al, Cu, and α-Fe. The range of temperatures was from -195 C to 0.8 temperature of melting point or phase change. The speeds varied from 6.6×10^{-4} to 2.1×10 per cent elongation per sec. The specimens had a diameter from 0.83 to 1.98 mm. It was found that the stress-strain diagram has two typical shapes: a "low temperature" in which failure occurs when, or soon after, the stress has reached its maximum value, and a "high temperature" in which the stress slowly decreases after reaching its maximum value. The speed of elongation has no definite influence on the shape of the stress-strain diagram. Sometimes (for β-Sn) the increase of speed changed the shape of the diagram from transient to high temperature, whereas the same change of speed caused the stress-strain diagram of cadmium to change from high-temperature to low-temperature shape. A further object of the experiments was to explore the phenomena which take place in the specimen after the stress has reached its maximum value in the stress-strain diagram of the high-temperature type. It was found that at the moment the stress has reached its maximum value the specimen begins to neck down at some point, then it stops necking down at this point and a neck begins to form at another place. This process continues over the whole length of the specimen, and is accompanied by a slow decrease of stress until a stable neck is formed at some point and the specimen breaks.

M. M. Gololobov, Czechoslovakia

893. Pierre Sorin, On the determination of energy dissipation by internal friction (in French), C. R. Acad. Sci. Paris 229, no. 22, 1123–1124 (1949).

Rolland and Koracs [C. R. Acad. Sci. Paris 227 (1948), p. 37 and p. 1019] have shown that the dissipated energy $\Delta E/E$ of a test piece may be determined by coupling two identical oscillating systems (pendulums) by means of that piece, and measuring the number of cycles N between two rest states and the logarithmic decrements of the normal modes. The author shows the following way to determine $\Delta E/E$: If the test piece has internal friction, the rest state of a pendulum is not well defined; only a minimum with small oscillations occurs. But it is possible to obtain a sharp rest state of the "start" pendulum by increasing its initial energy by a quantity ΔU. It is possible to choose ΔU so that the rest state occurs after $N/2$ cycles. For this case a calculation (not presented in the paper) for internal friction of the viscous type has given $\Delta E/E = 2\Delta U/N$. But this formula may have a general significance. The results obtained are in good agreement with that obtained by the methods mentioned above.

Albert Kochendörfer, Germany

Failure, Mechanics of Solid State

(See also Revs. 867, 871, 890, 892, 900, 993, 997)

894. E. Siebel and K. H. Bussmann, The notch problem for oscillating load (in German), Technik 3, 249–252 (June 1948).

A method of evaluation of the effect of notches on the behavior of structures under cyclic loading is presented. This differs significantly from other approaches in that not only the stress intensity at the bottom of the notch (point of maximum stress), but also the relative stress gradient at that point is taken into consideration, the latter being defined as the ratio of the change in stress per unit length to the maximum stress.

For smooth bars, the significance of the stress gradient is demonstrated by the observation that the endurance strength is a minimum for push-pull tests (zero stress gradient) and increases with decreasing bar size in reversed-bending tests (the straight-line gradient between maximum positive and negative stresses at the extreme fibers gets steeper as the diameter is reduced).

For notched specimens, the stress gradient varies sharply across the cross section; but, since failure is initiated in a narrow zone at the notch surface, only the local gradient there is of significance. Referring to Neuber's [*Kerbspannungslehre*, Springer,

Berlin, 1937] development of shape coefficients and stress-gradient characteristics for the extreme cases of (1) shallow circular surface notches in an infinitely wide flat bar, and (2) infinitely deep notches, the author proceeds to develop an approach by which both the stress-intensification factor and the significant stress gradient can be determined for the practically useful case of a notch of finite size in a plate of finite width, which then can be used to predict the useful life of a notched structure from endurance strength data obtained on smooth bars in reversed-bending tests producing equal, though uniform, stress gradients.

A. R. C. Markl, USA

Design Factors, Meaning of Material Tests

(See Revs. 873, 904, 907)

Material Test Techniques

(See also Revs. 873, 877, 886, 904, 907, 908)

895. **Albert Laborde and Maurice Leblanc, Dynamometric tracing of elasticity curves in testing textiles** (in French), C. R. Acad. Sci. Paris 226, 552–553 (Feb. 1948).

The author points out that the usual mechanical testing techniques of wires keep constant either the rate of increase of load or the rate of increase of deformation. When the instantaneous modulus of elasticity of a material changes appreciably, the increase in load or deformation can then be so sudden that the behavior of the material is influenced by the different speed in the application of the load or deformation. The author suggests an experimental arrangement to link linearly deformation speeds and load speeds, minimizing in that way any of their excessive increases.

A. J. Durelli, USA

896. **E. A. Finney and W. O. Fremont, Progress report on load deflection tests dealing with length and size of dowels,** Proc. Highw. Res. Bd., 27 Ann. Meet., 52–63 (1947).

This paper deals with measurements of relative displacements between two slabs connected by a load-transfer device (dowel) and subjected to shear loads. The authors designed a loading bridge allowing the measurement of the displacements by means of dial gages, and conducted a comparative test for dowels of several diameters and embedded lengths. The spacing between the slabs was $1/2$ in. and 1 in. The notion of joint modulus measured in pounds of load per inch of displacement is introduced to express the rigidity of the dowel. The authors feel that the method developed by them may be used to prepare specifications for load-transfer devices.

A. J. Durelli, USA

897. **Jacques Pomey, Louis Abel and François Goutel, On the determination of the elastic limit by Hertz's hardness measurement** (in French), C. R. Acad. Sci. Paris 228, 1478–1480 (May 1949).

Formulas are given to compute the state of stress for ball indentation in preloaded materials. [See following review, and also Rev. 2, **1114**].

J. A. Haringx, Holland

898. **Jacques Pomey, Louis Abel and François Goutel, Measurements of residual stresses on the surface of can-hardened and tempered steels** (in French), C. R. Acad. Sci. Paris 228, 1565–1567 (May 1949).

The determination of the resistance of a material against the indentation of a ball by means of the (electric) contact resistance is described. At a certain load the latter appears to fall down abruptly. The corresponding Hertz "hardness," defined as the (computed) maximum stress in the center of the contact area, is a function of the residual stresses present in the surface layer. It is linearly increased by compressive stresses and decreased by tensile stresses as shown by measurements on preloaded samples. [See preceding review.]

J. A. Haringx, Holland

899. **J. Pomey, L. Abel and F. Goutel, Determination of the intrinsic elastic limit by Hertz's hardness measurement** (in French), Génie Civ. 76, no. 12, 129–130 (June 1949).

The authors present an amplification of a former article [see Rev. 2, **1114**] in order to illustrate the manner of solving the problem previously presented. [See also following review.]

Frank J. Mehringer, USA

900. **J. Pomey, L. Abel, and F. Goutel, Development, effects, and measurements of superficial prestresses in machine parts** (in French), Génie Civ. 126, no. 5, 90–91 (1949).

This article is an abstract of a larger paper which discusses the effort of the authors to determine by means of ball impressions the effectiveness of case-hardening of machine parts. The ball impressions were made similar to that of a Brinell-hardness indentation. The authors use the Hertzian equations to determine the distribution of stress under the ball, and have interpreted their results by means of an envelope curve covering the Mohr's circles obtained for the stress conditions under various amounts of indentation.

R. G. Sturm, USA

901. **E. N. da C. Andrade and A. J. Kennedy, An automatic recording apparatus for the study of flow and recovery in metals,** Proc. phys. Soc. Lond. Sec. B, 62, Part 11, no. 359B, 669–675 (1949).

A complete description is given of an apparatus which records continuously on photographic paper the extension-time curve of a wire creeping under stress. The equipment consists essentially of a recording drum driven by motor that is controlled by a tuning fork, and of an optical system which projects onto the recording drum (a) timing lines, (b) a continuously recorded displacement scale, and (c) a point of light reflected by a small steel sphere that is carried by the wire specimen. As the displacement and time scales are recorded continuously with the displacement, shrinkage of the paper does not affect the accuracy of the record. Displacements up to 10 cm can be read within 0.02 mm. The load can be automatically removed and restored at preset intervals of time.

Irwin Vigness, USA

902. **Marcel E. Prot, Fatigue test under progressive load. A new technique for material testing** (in French), Rev. Metall. 45, 481–489 (Dec. 1948).

The author discusses certain disadvantages of conventional fatigue-testing methods, notably the time required, and then proposes an alternative involving progressively increasing the test load on each specimen until failure occurs. The load on the first specimen is increased in steps of relatively short time duration, while the load on succeeding specimens is increased in steps of longer and longer duration. In general, the lower the rate of load increase, the longer a specimen will last, but at the same time the lower will be the final failure load attained.

The author then plots final load on the specimens against square root of a load-increase rate. Both theoretical and experimental results are included to show that if the latter is chosen as abscissa, the curve can be extrapolated practically as a straight line back to zero increase rate, where it intersects the ordinate scale at a load level corresponding to the endurance limit. This is done for a light alloy, a soft steel, and a hard steel.

All tests are commenced at a load below the endurance limit sought. Early high-rate-of-increase tests are employed to estimate approximately the endurance limit so that later tests can be commenced at a load level only slightly below this.

There was recorded, after the paper was published, some discussion as to whether the absolute levels of endurance limits as found herein differed, as a result of understressing, from the conventional values; the author, however, claimed that he had not found its effect to be appreciable. Charles W. Gadd, USA

Mechanical Properties of Specific Materials

(See also Revs. 873, 890, 892, 894, 895, 900)

903. Katsurô Yamamoto, Photoelastic study of phenolformaldehyde resin (in Japanese), Trans. Soc. mech. Engrs. Japan 13, no. 44, 94–104 (1947).

The photoelastic properties of phenolformaldehyde which is widely used as a model material for photoelastic experiments, but which cannot be applied at present to the precise analysis of stresses in the freezing method of three-dimensional stress analysis from want of stability of its properties, are investigated experimentally. The changes of its modulus of elasticity, photoelastic constants, creep properties due to heat treatment and temperature change are measured, and good conditions for stress freezing and methods of finding true values of stress by removing the effect of initial stresses in test pieces are investigated. Some theoretical considerations for the relations between the phenomena of stress freezing, creep, double refraction and the chemical structure of the resin are stated. T. Udoguchi, Japan

904. A. E. Johnson and H. J. Tapsell, A comparison of some carbon molybdenum steels on the basis of various creep limits, Proc. Instn. mech. Engrs. 159, no. 40, 165–172 (1948).

The paper contains comparisons of various short-time creep limits with long-time creep data for six varieties of carbon-manganese-molybdenum steels. The experiments reveal that there is close resemblance between the various creep limits for molybdenum steels and the corresponding comparisons disclosed by the authors' earlier work on carbon steels [Johnson and Tapsell, Proc. Inst. mech. Engrs. 153, p. 169 (1945)]. The diversity in creep characteristics of molybdenum-steels renders it impossible to define a short-time creep test which will effectively evaluate working stresses at temperatures between 500 and 600 C, giving the same assurance of long life of 10,000 to 100,000 hr for a specified permissible creep. S. K. Ghaswala, India

905. G. Wallgren, An experimental determination of the fatigue diagram for alclad sheet specimens with rivet holes, subjected to tension and compression, Roy. Swed. Air Board Rep. Transl. no. 9, 12 pp. (1948).

This is a report on extensive tension-compression fatigue tests on alclad sheet specimens 0.047 in. thick, and 1.06 in. wide. Swedish aluminum alloy Ml 12p/3 was used which is equivalent to Alcoa 24S-T4. The ultimate strength was 64,000 psi. S-N curves are presented for five values of mean tensile stress up to 44,900 psi. These curves are carried up to 10 million stress cycles. For this life, the fatigue strength was ±9900 psi (under zero mean stress) and ±7000 psi (under 32,100 psi mean tensile stress). To prevent buckling of the thin sheet specimens subjected to alternating stresses, the authors used a wooden stiffening device, and experimental evidence is presented to show that this device did not affect the results.

Alexander Yorgiadis, USA

906. S. Luthander and G. Wallgren, An experimental determination of the fatigue diagram for tension and compression in alclad sheet, Roy. Swed. Air Board Rep. Transl. no. 8, 11 pp. (1948).

This is a continuation of the work covered by the preceding review. Specimens from the same aluminum sheet were subjected to the same tests, except that in this case each specimen (1.06 in. wide) had two holes 0.157 in. diam across its test section. S-N curves are presented up to 10 million cycles for mean tensile stresses ranging from zero to 32,100 psi. The holes reduced the static strength to 59,800 psi, and the fatigue strength at 10 million cycles to ±7,800 psi (under zero mean stress) and to ±4,300 psi (under 32,100 psi mean tensile stress). These stress values are based on the net stressed area. The authors use the above data to present curves of notch sensitivity as a function of mean stress and fatigue life. Values are also given for effect of surface finish. Alexander Yorgiadis, USA

907. G. Wallgren, Fatigue tests with stress cycles of varying amplitude, Flygtekn. Försöksanst. Medd. Rep. no. 28, 34 pp. (1949).

The paper presents the results of an extensive investigation of the effect of varying the amplitude of the stress cycles on the fatigue strength of 24S-T, 75S-T and Cr-Mo steel. The aim of the investigation is to check the cumulative-damage theory [M. A. Miner, Cumulative damage in fatigue, J. app. Mech. 12, 159–164 (1945)].

Two variations in the stress cycle or load spectrum were run: to simulate gust-load and maneuver-load conditions. In both cases, the load spectrum was based on work by Kaul [Die erforderliche Zeit- und Dauerfestigkeit von Flugzeugtragwerken, Jahrbuch 1938 der deutschen Luftfahrtforschung, 1, 274–288; Messergebnisse über Betriebsbeanspruchungen von Flugzeugflügeln, Luftwissensch. 1941, 8, no. 1, 20–24 (1941)].

The fatigue tests were performed on double-shear riveted joints of alclad 24S-T; unnotched specimen of alclad 75S-T; butt joint of alclad 75S-T; specimens of Cr-Mo steel with drilled hole; and unnotched specimen of alclad 24S-T.

The author found the values of the cumulative damage factor to range from 0.5 to 3.0 depending upon the material, type of specimen, and the load spectrum. The lowest values occurred in all cases under the load spectrum simulating gust loads.

Frank J. Mehringer, USA

908. E. P. Klier, F. C. Wagner and M. Gensamer, Fracture characteristics of ship plate in certain small-scale tests, Weld. Res. Suppl. 14, 50–66 (Feb. 1949).

This paper is another of a series which describes small-scale laboratory tests designed to indicate the fracture characteristics of large notched flat-plate tensile specimens, a prime objective being to develop small-scale tests which will predict brittle behavior of steels in ships. Two types of test specimens were used by the authors; a Schnadt-type bar with a 0.01-in. radius milled notch or a 0.0015-in. radius pressed notch was used for a slow-bend test, and a flat notched specimen was used for a tensile test. The specimens were of such dimensions that they could be fractured by a 60,000-lb testing machine. They were immersed in a bath of water or acetone to control their temperature. For the bend test, observations were made principally of the amount of contraction at the pin edge of the specimen, and of the per-cent ductile fracture as indicated by the appearance of the fracture surface. Observations of energy absorption, elongation, lateral contraction, fracture surface type, yield strength and tensile strength were made for the tensile specimens.

These observations are shown plotted as functions of tempera-

ture. Curves for the lateral contraction and the per-cent ductile fracture surface give the most sharply defined range of transition temperatures. A comparison is made of the maximum value of the transition temperature range for a variety of steels with the transition temperature of large (72 in. wide) flat-notched tensile specimens. It is concluded that a satisfactory correlation exists for the slow-bend tests for the steels tested.

For the edge-notched tension tests two transition temperature ranges were observed, one associated with the change of fracture type and another with the drop off in energy absorption. The transition temperature determined by fracture appearance (per-cent ductile fracture) is in approximate agreement with the 72-in. wide plate tests, while that determined from the energy absorption is not. Irwin Vigness, USA

Mechanics of Forming and Cutting

909. **F. F. P. Bisacre and G. H. Bisacre, The life of carbide-tipped turning tools,** Proc. Instn. mech. Engrs. 157, no. 35, 452–469 (1948).

This paper is a contribution towards a rational quantitative theory of the tool-life problem.

In the first part, the authors discuss an adiabatic theory of high-speed metal cutting, with no coolant. In machining medium-carbon steel with dry tungsten-carbide, cutting takes place under two widely different physical conditions, depending on the speed of cutting.

If the speed of cutting is high, there is not sufficient time for effective diffusion of heat to take place into the colder parts of the cutting; the heat in fact stays where it is generated. The conditions are adiabatic. On the other hand, when cutting is slow enough, there is time for complete diffusion of heat to take place, whereby the temperature becomes uniform through the thickness of the chip. This is the isothermal case.

It is found that these extremes occur when what the authors call Reynolds thermal number (speed $\times$ thickness of chip $\div$ thermal diffusivity) is large or small, respectively.

In the second part of the paper, a series of experiments made by M. F. Judkins and W. E. Uecker in 1933 on tool life is examined, and a rule is given that embodies the results of these tests. The limitations of this rule are pointed out.

In the third and last part of the paper, a study is made of those properties of the metal cut and of the tool that affect the life of the tool. The method of dimensional analysis is applied to deduce a general form for the life-law. Judkins and Uecker's experimental result conforms to this law within the limits of the tests. But further experiment alone can confirm the general form—in its wider application. Several discussions and communications on the subject are published at the end of the paper.

From authors' summary by A. J. Durelli, USA

Hydraulics; Cavitation; Transport

(See also Revs. 919, 923, 976)

910. **Charles W. Harris, The influence of random roughness on flow in pipes,** Bull. Univ. Wash. Engng. Exp. Sta. no. 115, 17 pp. (1949).

Experimentation on artifically roughened pipes varying from 4 in. to 1/4 in. diam is reported upon, as well as tests on a rusted 2-in. black iron pipe. The artificial roughnesses were produced by spraying stainless steel on no. 20 sheet steel, then forming the plates as inserts into supporting pipe. A novel tilting-mirror gage was employed for the measurement of small pressure drops.

The results for the artificially roughened pipes indicated a complete lack of transition from smooth-pipe friction to fully developed boundary roughness, i.e., the friction-factor Reynolds-number chart followed the smooth-pipe curve for smaller Reynolds numbers, then left the curve and followed a horizontal line for larger Reynolds numbers. It was also found that the transition from the smooth curve to the fully developed curve for natural pipes could be reproduced by assembling in series sections of pipe of varying roughness.

A report on this work was previously published in the ASCE Proceedings of Spring 1949. Victor L. Streeter, USA

911. **V. V. Vedernikov, Experiments on hydraulic pumps (pump as a transitory region)** (in Russian), Doklady Akad. Nauk SSSR 59, no. 3, 447–450 (1948).

Experimental investigations on the hydraulic jump generated by the flow over a spillway lead to the conclusions: (1) The length of the jump, defined as the distance within which a normal velocity distribution is reached, is of the order of 40 times the water depth. (2) In the intermediate sections the flow is characterized by two longitudinal vortexes located between the vertical center plane and the side walls of the water channel. These vortexes influence decisively the velocity distribution and the erosive action of the flow. G. Weinblum, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 910, 954, 965, 973, 1001)

912. **R. S. Rivlin, The hydrodynamics of non-Newtonian fluids. II,** Proc. Cambridge philos. Soc. 45, 88–91 (1949).

In part I [Proc. roy. Soc. London Ser. A, 193, 260–281 (1948); Rev. 2, **1024**] the author established general equations of motion for a non-Newtonian fluid. In this paper these equations are applied to the flow of such a fluid through a cylindrical tube of circular cross section. It is found that the axial normal stress σ_{zz} is no longer uniformly distributed over the cross section of the tube, as in the case of a Newtonian fluid. However, the derivative $\partial\sigma_{zz}/\partial z$ is constant throughout the tube. W. Prager, USA

913. **Godofredo Garcia, Exact, canonical and basic formulas for internal stresses and viscous flow** (in French), Ann. Soc. Polon. Math. 21, 107–113 (1948).

The two-dimensional formulation of equations given by the author [Actas Acad. Ci. Lima 10, 117–170 (1947)]. The notations are not clear. L. M. Milne-Thomson, England

914. **Robert Miche, On the reduction to a variational principle of not necessarily slow flows** (in French), J. Math. pures appl. 28, 151–179 (1949).

The author first defines a volume integral over a fixed domain D: $\Phi = \int\int\int L(\psi^\alpha)d\tau$, where L is the Lagrangian. In the case of steady motion, L is a function of vector functions Ψ^α and their partial derivatives of order n. The necessary condition for an extremum leads to Euler's equations

$$E^i(L) = \frac{\partial L}{\partial \psi^\alpha} - \frac{\partial}{\partial x_\alpha}\frac{\partial L}{\partial \psi^i{}_\alpha} + \frac{\partial^2}{\partial x_\alpha \partial x_\beta}\frac{\partial L}{\partial \psi^i{}_{\alpha\beta}} - \cdots = 0 \quad (1).$$

Here x_α denotes a vector, while the subscripts of the vector ψ^i denote partial differentiation. If the motion is unsteady, the vector functions ψ^i will depend on x_α as well as the time t. The Lagrangian will contain terms involving the first partial derivatives with

respect to t. The condition for an extremum is then subject to three additional equations

$$E^{\alpha}(L) = \frac{\partial L}{\partial \psi^{\alpha}{}_{t}} - \frac{\partial}{\partial x_{\alpha}} \frac{\partial L}{\partial \psi^{\alpha}{}_{\alpha t}} + \cdots = 0 \qquad (2).$$

As is well known, the problem is essentially indeterminate. In the case of steady motion, there are seven unknown functions ψ^{α}, λ^{α} and L for only six equations. On the other hand, if the motion is unsteady, there will be more equations than there are unknowns. The author solves this difficulty by choosing the Lagrangian to be the sum of the rate of increase of kinetic energy per unit volume T and that of the dissipation per unit volume 2Ψ. Equations (2) then serve to determine λ^{α} while equations (1), which correspond to the Navier-Stokes equations, are consequences of the principle of conservation of energy. The solution is verified to be consistent. In the special case of steady motion, $T = 0$, $L = 2\Psi$. This constitutes the well-known Helmholtz theorem. Some other special cases are also discussed.

Finally, the author makes the conjecture that his complete Lagrangian is a divergence, because in that case Euler's equations will be identically satisfied. This is actually verified in the two-dimensional case. Y. H. Kuo, USA

915. L. Castoldi, Bernoulli's surfaces and lines in steady motion of a real fluid (in Italian), Atti Accad. Ligure 4, 21–25 (1948).

In extension of Bernouilli's theorem, Lamb [Proc. London Math. Soc. (1) 9, 91–92 (1878)] proved that in any steady motion with velocity V, if an acceleration potential W exists, then it follows that the quantity $V^2/2 + W$ has a constant value on each surface everywhere tangent to the vortex lines and the streamlines. The author attempts to formulate a corresponding theorem for barotropic motions of a viscous fluid of constant viscosity, subject to conservative extraneous force. Writing the dynamical equation in the form curl $V \times V + \nu$ curl curl V = grad Φ, he proves that Φ = constant on each vortex line and streamline if and only if $(\nu V \times \text{curl } V) \times \text{curl curl } V = 0$. He gives examples to show that motions satisfying his conditions exist. The brackets in his formula (2) are a misprint.

C. A. Truesdell, USA

916. G. I. Taylor and R. M. Davies, The aerodynamics of porous sheets, Rep. Memo. aero. Res. Counc. Lond. no. 2237, 14 pp. (1944, publ. 1947).

Measurements of the air resistance of perforated sheets, wire gauzes and some fabrics are described.

From pressure-difference measurements a resistance coefficient $K = (P_1 - P_2)/\frac{1}{2}U^2$ is derived, where U is the main velocity of a stream through a porous sheet placed at right angles to the stream. A skeleton formula is given for K, in terms of a contraction coefficient C and a recovery coefficient. Further, drag coefficients $C_D = R/\frac{1}{2}V^2$ were found by dropping sheets and gauzes.

The theoretical relation $C_D = K/(1 + K/4)^2$ [same source, no. 2236; Rev. 3, **730**] is well supported by the present experiments as long as K is not too large, while earlier arbitrary theories for K do not agree with observations. Geometrical parameters for describing the porosity are discussed.

Georg P. Weinblum, USA

917. Johannes Weissinger, On a generalization of the Prandtl lifting-line theory (in German), Math. Nachr. 2, 45–106 (1949).

[This is a detailed report on the author's work which was first reported in Germany in 1942 and has since become familiar to engineers in Nat. adv. Comm. Aero. tech. Memo. no. 1120 (1947); Rev. 1, **509**]. For a thin lifting surface in a uniform stream of an ideal incompressible fluid, an integrodifferential equation can be written for the bound-vortex strength distribution over the surface, satisfying the given slope conditions at points of the surface. This equation is put into a form where it appears as the well-known Prandtl lifting-line equation with a correction term in the induced velocity. This correction term is now calculated by an approximate method. This is the author's "F (Fläche)-method"; it amounts to determining the whole induction effect by means of the induced downwash at the $\frac{3}{4}$-chord line, this downwash being calculated for a standard chordwise, but arbitrary spanwise, bound-vortex distribution. A numerical method for routine use of this procedure is worked out, but because it requires excessive labor, a further simplification is made in the "L (Linie)-method." Here the idea of a chordwise distribution of vortexes is discarded, and the bound- and trailing-vortex effects are evaluated for a single concentrated lifting line at the $\frac{1}{4}$-chord line. Again these effects are evaluated at the $\frac{3}{4}$-chord points to determine their effect on circulation. This method is no more difficult to employ than the customary procedures developed for the Prandtl theory, provided certain numerical tables are at hand; it has the great advantage of being able to handle yawed and sweptback wings.

The author's numerical results show the L-method to be a close approximation to the F-method in practical cases. In a footnote it is mentioned that the L-method is the same as Mutterperl's [Nat. adv. Comm. Aero. tech. Note no. 834 (1941)], except for the numerical treatment. Tables of numerical constants for use with the F- and L-methods are given. In the numerical formulation, use is made of a numerical-quadrature formula of Multhopp [Luftfahrtforschung 15, 153–169 (1938)]. In a mathematical supplement the author considers the extension of his methods to various classes of plan forms, the properties of certain functions, and other details. [For another recent paper on this subject, see E. Reissner, Proc. nat. Acad. Sci. 35, 208–215 (1949)].

W. R. Sears, USA

918. V. V. Golubev, On the theory of a wing of small span (in Russian), Uchen. Zapiski Moskov. Gos. Univ. Mekh. 122, 3–16 (1948).

It is well known that for aspect ratios less than about unity the agreement between experimental and theoretical lift distribution based on the lifting-line theory of Prandtl breaks down. The paper under review presents a modification of the conventional theory by assuming that the free-vortex sheet does not leave the wing at the trailing edge but at some line of the suction side at a distance approximately one quarter of the chord length from the leading edge and in the same plane as the main stream. The wing itself is supposed to be replaced by a plane. The first assumption leads to a secondary stream leaving the wing at the tips. The secondary stream, contrary to the main stream, is not supposed to be subjected to Joukowsky's hypothesis. The presence of the secondary stream, in the case of elliptic loading, produces an increase of circulation proportional to the square of the angle of attack θ and

$$\varphi(\chi, \theta) = \frac{2\lambda}{\pi + 2\lambda}\left[1 + \pi\sqrt{3}\,\frac{\theta}{\pi + 2\lambda}\right] \qquad (1)$$

gives the increase of the lift L as compared with a two-dimensional motion for given aspect ratio λ and angle of attack θ. Instead of a linear relation as given by the lifting line theory, a quadratic relation between the lift coefficient L_e and the angle of attack θ is set up. The main results of the paper may be summed up as follows: (a) The values of L_e calculated by the conventional theory are too small; a better approximation of the ob-

served values is given by formula (1). (b) For very small aspect ratios ($\lambda < 1$), the critical angle of attack increases rapidly. For $\frac{1}{2} < \lambda < 1$, the measured values for the critical angle of attack are higher than the calculated ones [cf. Golubev, Izv. Akad. Nauk SSSR 1947, 261–270]. E. Leimanis, Canada

919. **E. Maurer, The critical Reynolds number for circular tubes by the entropy principle** (in German), Z. Phys. 126, 522–532 (1949).

An entropy analysis is made of the flow of an incompressible fluid or an ideal gas through a smooth cylindrical tube. This analysis shows a critical Reynolds number (below which laminar flow is stable) of 1440. The experimentally determined value for this critical Reynolds number is about 2000.

R. C. Binder, USA

920. **James W. Beach, Flow of viscous fluid between slowly rotating eccentric cylinders,** Iowa State Coll. J. Sci. 23, 7–10 (1948).

This paper deals with the problem of the slow, steady motion of an incompressible viscous fluid between two eccentric, rotating, infinitely long cylinders. As the stream function satisfies a biharmonic equation, the general solution can be expressed in terms of analytic functions of a complex variable. The author then maps the region between the eccentric circles on the region between concentric circles and solves the boundary-value problem in the transformed plane. The force and torque on either cylinder are given and discussed. Of course, by different methods this problem has already been treated by many authors, for instance G. B. Jeffery [see H. L. Dryden, F. D. Murnaghan and H. Bateman, Bull. Nat. Res. Counc. no. 84, 232–242 (1932)].

Y. H. Kuo, USA

921. **G. Szegö, The virtual mass of nearly spherical solids,** Duke Math. J. 16, 209–223 (1949).

An incompressible, inviscid fluid fills the whole space outside of a rigid body S, and flows without vorticity. Its velocity is zero at infinity, and S is translated with a velocity U. If T is the energy of the fluid, $W = T/\frac{1}{2}U^2$ is the virtual mass of S, and W_m its mean value over all the directions of the velocity U. Pólya [Proc. nat. Acad. Sci. Wash. 33, 218–281 (1949)] treated the problem for ellipsoids and infinite cylinders, while the author considers an almost spherical surface $r = 1 + \delta f(\theta,\varphi)$. The expression for W_m, if powers of δ higher than the second are neglected, proves that the minimum is reached for a sphere, as conjectured by Pólya. Some other quantities similar to W_m, and certain considerations on the cylindrical case complete the paper.

Courtesy of Mathematical Reviews M. Brelot, France

922. **J. J. Koch, Experimental method for determining the virtual mass of oscillations of ships,** David Taylor Model Basin Transl. 225, 9 pp. (1949).

The investigation reported in this paper was performed to explain discrepancies between measured (97–98 cpm) and calculated (152 cpm) frequencies (based on the ship's true mass). The author assumes an ideal fluid, plane motion, and potential flow. He writes the two equations of the motion, the equation of continuity, the condition of zero vorticity, and seeks the boundary conditions for the pressure.

He then considers a two-dimensional electric-current field in a homogeneous plate of constant thickness. Because of the complete analogy between the equations of the two problems, the hydrodynamic problem can be solved by the electrical analogy provided the proper boundary conditions are applied. The author then determines the expression of the virtual mass for the hydrodynamic problem and the corresponding expression for the electrical problem. The different elements of this last expression are measured in the electrical model. A renewed computation based on the experiments discussed in the paper furnishes a value of 96.5 cpm for the calculated frequencies (based on the virtual) —a very satisfactory agreement with the measured frequencies.

L. J. Tison, Belgium

923. **Manuel L. Acevedo, Cavitation in marine propellers and its forecast by means of tests with profiles** (in Spanish), Canal exp. hidrodin. El Pardo, 36 pp. (1948).

The condition of no cavitation for blades of marine propellers is established from tests of pressure distribution over circular-arc airfoils and wing profiles, made by Gutsche, and the corrections for cascade effect are put in account.

Giulio de Marchi, Italy

924. **N. K. Kalinin, On the instability of filtration in the case of a drain in a water-permeable layer of finite depth** (in Russian), Prikl. Mat. Mekh. 12, 199–206 (1948).

The same problem on the lowering of the water table as that of an earlier paper [N. K. Kalinin and P. J. Polubarinova-Kochina, Prikl. Mat. Mekh. 11, 231–236 (1947); Rev. 1, **307**] is considered here with the only difference that the drain is now not located in a semi-infinite plane, but at the bottom of a layer of unit thickness.

Courtesy of Mathematical Reviews H. P. Thielman, USA

925. **Garbis H. Keulegan, Interfacial instability and mixing in stratified flows,** J. Res. nat. Bur. Stands. 43, no. 5, 487–500 (Nov. 1949).

When a liquid flows over a body of a heavier liquid, with which it is miscible, an interface may be formed at which there is a sharp discontinuity of densities but not necessarily of velocities. As the relative velocity is increased, waves are formed at the interface, and at a critical velocity mixing begins by the formation of eddies periodically ejected from the crests of the waves. The critical values of the parameter (criterion of mixing)

$$\theta = (\nu g \Delta\rho/\rho_1)^{1/3}/U,$$

where ν is the kinematic viscosity, $\Delta\rho$ the difference of the air and water densities, ρ_1 the density of air and U the velocity of the air over the waves, were determined experimentally. The derivation of the criterion is theoretical and based on the work of Jeffreys [Proc. roy. Soc. vol. 107 (1925) and 110 (1926)]. Experimental data were also obtained on the relation of the amount of mixing to the relative velocity of the liquids at velocities higher than the critical. Author's summary

926. **R. S. Burdon, Surface tension and the spreading of liquids,** Cambridge University Press, Cambridge, 1949, xiv + 92 pp., 22 figs. Cloth, 9.7 × 5.7 in., $3.

The aim of this short monograph is to present an up-to-date survey of the subject of the title for nonexperts in this field. The present book is a revision of an earlier edition of 1940. It is written primarily from the physical chemist's point of view and overlaps relatively little with expositions of capillarity of the sort to be found in Gyemant's article in volume 7 of the *Handbuch der Physik*. In the reviewer's mind this is all to the good. No one who has read this monograph describing the various and complex phenomena associated with surface forces will be inclined to regard surface tension as a conceptually simple, though mathematically complicated, addition to hydromechanics.

Over a third of the book is devoted to phenomena associated

with the spreading of one liquid over another, and these are perhaps the most interesting sections. However, there are also chapters on methods of measuring surface tension (including a discussion of the difficulty in making the measurement for mercury) and on liquids in contact with solids. In the first chapter on the nature of surface forces, the author seems to adhere to the school believing that surface tension is a fiction and that surface energy is the reality. However, the difficulty seems to be semantic and not worth belaboring inasmuch as it has been recently treated elsewhere [e.g., Prandtl, Ann. Phys. 1, 59–64 (1947); Rev. 2, 195]. The exposition is clear and understandable, even to one (like the reviewer) with only an elementary knowledge of chemistry, and should be instructive to anyone interested in the mechanics of liquids. J. V. Wehausen, USA

Compressible Flow, Gas Dynamics

(See also Revs. 957, 964, 969, 971, 973, 974, 975)

927. Caius Jacob, On the influence of compressibility in fluid flow, Acad. Roum. Bull. Sect. Sci. 28, 637–641 (1946) = Disquisit. Math. Phys. 6, 193–223 (1948).

This paper seems to be a new edition of an older one [Acad. Roum. Bull. Sect. Sci. 28, 637–641 (1946)] in which the author, following E. Lamla [Luftfahrtforschung 19, 358–362 (1943)], writes the complex potential of the flow as

$$f(z,\bar{z}) = \varphi(x,y) + i\psi(x,y) = f_0(z) + M^2 f_1(z,\bar{z}),$$

where $z = x + iy$, $\bar{z} = x - iy$ and M is the characteristic Mach number of the flow. The function f_1 is written as

$$f_1(z,\bar{z}) = \tfrac{1}{4}U^{-2}(df_0/dz)\int\overline{(df_0/dz)^2}dz + \tfrac{1}{4}[g_1(z) - f_0(z)],$$

where U is the characteristic velocity of the problem. Applying this method to the problem of free jets produced by two walls symmetrically situated with respect to the x-axis, the author takes U as the velocity at the free jet boundary; M is the Mach number corresponding to U. Thus f_0 is the incompressible solution. The calculation of the compressible solution then reduces to the determination of g_1. The author states that with previous results on liquid jets obtained by A. Weinstein, G. Hamel, H. Weyl, K. Friedrichs and J. Leray, g_1 usually can be uniquely determined up to a constant by the boundary conditions and by solving a Fredholm integral equation. This constant is fixed by the condition at the points of separation. When the walls are straight, the author obtains the simple result for the contraction factor of the jet as $\Gamma = \Gamma^0 1 + \frac{1}{2}M^2(1 - \Gamma^0)$, where Γ^0 is the contraction factor for incompressible flow. H. S. Tsien, USA

928. Ferdinand Cap, On two procedures for one-dimensional nonstationary gas dynamic problems (in German), Acta Physica Austriaca 2, 224–238 (1949).

Two procedures are presented for calculating the unsteady one-dimensional motion of a gas under the assumption of isentropy. One of these is the method of characteristics of Riemann derived by means of the Legendre transformation. The other is the graphical method of F. Schultz-Grunow [Forsch. Geb. Ing.-wesens 13, 125–134 (1942)], which is described and extended. The application of this method to nonadiabatic flow processes is indicated. W. D. Hayes, USA

929. Manfred Schafer and W. Tollmien, Two-dimensional potential flows, Nat. adv. Comm. Aero. tech. Memo. no. 1243, 24 pp. (1949).

The characteristic differential equations for two-dimensional compressible potential flow are set up. They are integrated in a form containing the magnitude of the velocity and the deflection angle only. The methods of Walchner, Prandtl, and Busemann are discussed and examples are given. Tables are given for deflection angle, pressure, velocity, Mach number, and Mach angles for isentropic changes with $\gamma = 1.405$.

H. P. Liepman, USA

930. Max A. Heaslet and Harvard Lomax, The use of source-sink and doublet distributions extended to the solution of boundary-value problems in supersonic flow, Nat. adv. Comm. Aero. Rep. no. 900, 15 pp. (1948).

See Rev. 1, 500.

931. S. Goldstein, M. J. Lighthill, and J. W. Craggs, On the hodograph transformation for high-speed flow. I. A flow without circulation, Quart. J. Mech. appl. Math. 1, 344–357 (1948).

In a previous paper [Proc. roy. Soc. London Ser. A, 191, 323–341 (1947); Rev. 1, **994**] Lighthill indicated a method of constructing (in the hodograph plane) steady two-dimensional flows past closed profiles which reduce to given incompressible flows for vanishing free-stream Mach number [cf. also T. M. Cherry, ibid. 192, 45–79 (1947); Rev. 1, **870**]. An alternative method is presented in this paper for the case of a circulation-free flow "corresponding" to an incompressible flow past a circle. The authors start with the remark that for this incompressible flow the complex potential as a (multiple-valued) function of the conjugate complex velocity ζ is represented by four series w_1, w_2, w_3, w_4 in different parts of the (two-sheeted) hodograph; and that the analytic continuation of w_1 leading to w_2, w_3, w_4 may be obtained by considering the integral

$$(1/2\pi i)\int_{-\infty i}^{+\infty i} f(v)(-\zeta)^v dv.$$

For a compressible fluid they begin with a solution of Chaplygin's well-known equation of the form $\psi = \mathrm{Im}W_1$,

$$W_1 = \Sigma_0^\infty a_n\psi_n(\tau)e^{-n(s_1+\theta i)}$$

Here τ, θ are Chaplygin's hodograph variables,

$$\psi_n(\tau) = \tau^{n/2}F_n(\tau)$$

Chaplygin's solutions, and s_1 a parameter depending on the stream Mach number. The analytic continuation of W_1 is achieved by means of the integral

$$(1/2\pi i)\int_{-\infty i}^{+\infty i} f(v)(-1)^v\psi_v(\tau)e^{-v(s_1+i\theta)}\,dv.$$

The final solution has the four representations $W_1 + L$, $W_2 - W_1 - L$ and $-W_2$, where

$$W_2 = i\sum_0^\infty g_n\psi_{(1/2)-n}(\tau)e^{(n-1/2)(s_1+i\theta)}$$

$$L = \sum_2^\infty h_nC_n\psi_n(\tau)e^{n(s_1+i\theta)};$$

the g_n, h_n, C_n are numerical constants. The series for W_2 converges for $\tau > \tau_1$ (τ_1 the free-stream speed), the series for L everywhere. Possible generalizations of this method, and connections with Lighthill's general solution, are discussed.

L. Bers, USA

932. M. J. Lighthill, On the hodograph transformation for high-speed flow. II. A flow with circulation, Quart. J. Mech. appl. Math. 1, 442–450 (1948).

This is a continuation of a previous paper by the author, Goldstein, and Craggs [see preceding review]. This time it is assumed that the basic incompressible flow past a circle has nonvanishing circulation. If the development of part I is followed, one obtains a stream function which is multiple-valued in the physical plane. A modification of the method, however, overcomes this difficulty. The author concludes that the present method is inferior to the one developed by him previously [Proc. roy. Soc. London, Ser. A, 191, 323–341, 341–351, 352–369 (1947); Rev. 1, **994**].

L. Bers, USA

933. **R. K. Tempest and L. Rosenhead, Notes on the linearised equation for the velocity potential of the steady supersonic flow of a compressible fluid,** Proc. London Math. Soc. 51, 197–212 (1949).

Various general solutions of the two-dimensional wave equation have been collected and interpreted with reference to the linearized equation for steady supersonic potential flow,

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 = \alpha^2\partial^2\phi/\partial z^2,$$

where $\alpha^2 = M^2 - 1 > 0$. They include, as special cases, linearized flows considered by numerous authors. For example, the von Kármán-Moore solution for a cone [Trans. ASME 54, 303–310 (1932)] is both a Legendre function and a Whittaker integral solution. In cylindrical or rectangular coordinates the search for homogeneous solutions $\phi = z^n\Phi(t)$, where $t = z/r\alpha$ or $z/y\alpha$, leads to expressions for $\Phi(t)$ in terms of hypergeometric functions. Some of these axisymmetric "cone" solutions were previously given by Hayes [Quart. appl. Math. 4, 255–261 (1946)]. By means of Gauss's relation for hypergeometric functions most of the plane "wedge" solutions can be transformed into the natural forms $A(z \pm \alpha y)^n$ for some constant A.

Courtesy of Mathematical Reviews J. H. Giese, USA

934. **W. N. Bailey, A note on the paper by Tempest and Rosenhead,** Proc. London Math. Soc. 51, 213–214 (1949).

By means of Kummer's formula for hypergeometric functions the rest of the plane solutions $z^n\Phi(z/y\alpha)$ found in the preceding paper have been transformed into the form

$$A[(z + \alpha y)^n \pm (\pm 1)^n(z - \alpha y)^n].$$

Courtesy of Mathematical Reviews J. H. Giese, USA

935. **Tables of supersonic flow around cones of large yaw, by the staff of the Computing Section, Center of Analysis, under the direction of Zdenek Kopal,** Massachusetts Institute of Technology, Department of Electrical Engineering, Center of Analysis, Technical Report no. 5. Cambridge, 1949, xviii + 125 pp. (1 plate).

These tables are based on unpublished calculations by A. H. Stone. In previous publications [same source, no. 3 (1947); also Stone, J. math. Physics **27**, 67–81 (1948); Rev. 1, **993**], the flow about a cone at a small angle of yaw ϵ was calculated by expanding in powers of ϵ and discarding $O(\epsilon^2)$. In his new work, Stone has evaluated the ϵ^2 terms, so that a second approximation is possible. Only the results used in numerical computations are reproduced here. It is found that the shock wave is, to the second approximation, conical, and is yawed to the same angle as predicted by the first-order theory. It is, however, not a circular cone, but is nearly elliptic. The head drag, which was unaffected by yaw in the first approximation, involves second-order corrections, but the normal force remains the same as in the first approximation. The tables permit all the interesting flow quantities for the region between the cone and the shock wave to be calculated, to the second order, for seven cones having semi-apex angles between 5 deg and 25 deg. In each case a range of Mach numbers is covered. W. R. Sears, USA

936. **Olga Taussky, A boundary value problem for a hyperbolic differential equation arising in the theory of the non-uniform supersonic motion of an aerofoil,** Courant Anniv. Vol., Interscience Publishers, New York, 1948, pp. 421–435.

The result of Possio [C. Possio, Acta Pont. Acad. Sci. I, 11 (1937)] concerning the oscillation of a thin strip in a supersonic flow is rederived. The method used here has the advantage that it establishes the uniqueness of the result. G. F. Carrier, USA

937. **L. K. Kudriashov, Plane parallel gas flow past an ellipse,** Hdqtrs. Air Mat. Comm. Dayton Transl. no. A9-T-25, 20 pp. (Jan. 1949).

See Rev. 1, **868**.

938. **G. Drougge, Wing sections with minimum drag at supersonic speeds,** Flygtekn. Försöksanst. Medd. Rep. no. 26, 15 pp. (1949).

Use is made of the Ackeret linearized two-dimensional supersonic theory to investigate the effect of several wing section parameters on pressure drag and also on total drag. The parameters considered are maximum thickness, section area, and section modulus. Both solid and shell sections are considered. The results of pressure drag are well known in the classified literature in the USA and undoubtedly also in the comparable literature in all countries in which supersonics has been of major interest. Sections found to have advantages are the double wedge, parabolic arcs, sine curves, and so-called "Γ-profiles." The bicircular profile is also considered because of the attention it has received in the past. The author found little variation in pressure-drag coefficients among the smooth, curved profiles which were generally found to be superior to the double wedge. This is especially true in the case of the shell sections.

Comparisons on total drag were made on the basis of drag per unit of parameter value. Drags were divided by the thickness, area, and section modulus respectively, and comparisons were made of the respective quotients. A minimum existed only in the case of drag per unit area. In the other cases, drag decreased with increase in parameter. Max G. Scherberg, USA

939. **Robert T. Jones, Subsonic flow over thin oblique airfoils at zero lift,** Nat. adv. Comm. Aero. Rep. no. 902, 6 pp. (1948).

The detailed theoretical study of the effect of thickness on the pressure of a three-dimensional airfoil at subsonic speeds is of comparatively recent date although special cases are included in results of classical hydrodynamics. Basically the velocity potential due to thickness of a given airfoil can be written down in terms of a surface distribution of sources of known strength, but in actual fact the evaluation and interpretation of the result may still require a considerable effort. The present author uses as fundamental solution a simple "source" for the pressure potential (i.e., the induced longitudinal velocity or, except for a constant of proportionality, the acceleration potential). In terms of the velocity potential, this corresponds to a semistraight line, or ray, extending downstream in the direction of flow. The source-sink pairs for the pressure potential mentioned by the author correspond to simple sources for the velocity potential. It is shown that an (oblique) line segment of sources for the pressure potential corresponds to a wedge-shaped airfoil. It produces pressures which can be interpreted in terms of cone-fields emanating from the tips. More general cases of airfoils, including sweptback wings, are then obtained by superposition. It is found that the pressure distribution on certain sweptback wings is continuous across the

speed of sound. At subsonic speeds, the spanwise drag is positive near the root section, but this is balanced by negative drag at the tips, as required by d'Alembert's paradox. [Compare F. Ursell, Flow round sweptback wings, Aero. Quart. 1 (May 1949), containing further references to selected work by Neumark, Küchemann and Ludwieg.] A. Robinson, England

940. Robert A. Spurr and H. Julian Allen, A theory of unstaggered airfoil cascades in compressible flow, Nat. adv. Comm. Aero. Rep. no. 888, 14 pp. (1947).

The performance of airfoils in unstaggered cascades is estimated from the corresponding performance of one of the airfoils alone. The method is to express the pressure coefficients in terms of the vorticity distribution along the chord. The vorticity is then expanded in Fourier series and coefficients equated up to the first order of $\sqrt{(1 - M^2)}$ for compressible effects, and the first order in the square of the chord-gap ratio. In this manner there are derived approximate relations between the cascade and free airfoil angle of attack, pressure coefficient, lift coefficient, and Mach number. These relations can then be used to compute the performance of a given cascade, or to produce a cascade for a given performance. Several worked examples show the simplicity and precision of the method. Howard W. Emmons, USA

941. George R. Costello, Methods of designing cascade blades with prescribed velocity distributions in compressible potential flows, Nat. adv. Comm. Aero. tech. Note no. 1970, 23 pp. (1949).

By the use of the assumption that the pressure-volume relation is linear, a solution to the problem of designing a cascade for a given turning and with a prescribed velocity distribution along the blade in a potential flow of a compressible perfect fluid, was obtained by a method of correspondence between potential flows of compressible and incompressible fluids. If the prescribed velocity distribution is not theoretically attainable, the method gives a way of modifying the distribution so as to obtain a physically significant blade shape. R. C. Binder, USA

942. L. G. Loĭtsyanskiĭ, Resistance of a grid of profiles in a gas stream with subcritical velocities (in Russian), Prikl. Mat. Mekh. 13, 171–186 (1949).

In a previous paper (see Rev. 1, **1140**) the author developed an approximate method for determining the reactions on a cascade in a viscous incompressible fluid in terms of the properties of the boundary layer at the trailing edge of the cascade. In the introduction to the present paper, the (unsupported) claim is made that these results have been "beautifully confirmed by experiments." The bulk of the paper consists of the step-by-step generalization of the previous paper to the case of compressible flows at subcritical Mach numbers. M. V. Morkovin, USA

943. Franklin W. Diederich and Martin Zlotnick, Theoretical spanwise lift distributions of low-aspect-ratio wings at speeds below and above the speed of sound, Nat. adv. Comm. Aero. tech. Note no. 1973, 21 pp. (1949).

The spanwise lift distribution over wings of small aspect ratio are calculated on the basis of the virtual-mass concept (cf. same source, tech. Rep. no. 835, by R. T. Jones) and the Weissinger *L*-method (same source, tech. Memo. no. 1120). The former approach is shown to imply that the lift depends entirely on the geometry of the widest section. The Weissinger method consists of concentrating the bound vorticity at the quarter-chord line, calculating the downwash at the three-quarter-chord line, and equating the downwash angle there to the slope of the mean-camber surface.

Both of the above approaches are shown to lead to essentially the same integral expression for the local loading coefficient. This expression is then used to calculate and plot numerical results for the spanwise lift distributions corresponding to several angle-of-attack distributions. These include symmetric and antisymmetric twists, as well as flap and aileron deflections.

A rather detailed discussion is given of the range of validity of the analysis, and it is shown that the aspect ratio must not have a value greater than unity. It is indicated that the results are valid for supersonic speeds, provided that the leading edge is swept back sufficiently far behind the Mach cone.

Morris Morduchow, USA

944. H. J. Stewart, The lost solutions in axially symmetric irrotational compressible fluid flow, Quart. appl. Math. 6, 334–337 (Oct. 1948).

It is shown that an axially symmetric potential gas-flow whose hodograph (in a meridian plane) is one-dimensional is either a flow due to a two-dimensional line source superimposed on a uniform flow, or the Taylor-MacColl flow past a cone.

L. Bers, USA

945. S. Luthander, G. Axell, and T. Gullstrand, Pressure-distribution tests and investigation of flow pattern for circular air intakes with various nose profiles, Roy. Swed. Air Board Rep. Transl. no. 4, 50 pp. (1949).

Wind-tunnel tests were conducted on 3 circular air intakes having different lip profiles over a range of angles of attack and a wide range of inlet-velocity ratios. Pressure distributions were taken along the top centerline on both the inside and outside surfaces. The tests were made at free-air speeds below 35 m/sec. No internal pressure-recovery measurements were made.

J. M. Wild, USA

946. John W. Miles, Errata: Aerodynamic forces on an oscillating flap at supersonic speeds, J. aero. Sci. 16, 442–443 (1949).

Using the general results developed in an earlier paper [same source, 14, 351–358 (1947); Rev. 1, **156**], the complex flutter coefficients are found. Extract from the paper

947. Antonio Ferri, Method for evaluating from shadow or schlieren photographs the pressure drag in two-dimensional or axially symmetrical flow phenomena with detached shock, Nat. adv. Comm. Aero. tech. Note no. 1808, 1–14 (Feb. 1949).

A procedure is outlined whereby the pressure drag of a body with a detached bow wave can be determined when the shape and location of the wave are known (as from optical observations). The procedure, which is applicable in either the planar or axially symmetric case, is based on the method of characteristics for rotational supersonic flow. The characteristics construction is begun from a length of bow wave outboard of the localized subsonic region and is continued in toward the body until conditions are determined along a characteristic line extending the full distance from the wave to the body. (The tacit assumption is made that this construction can be accomplished by using a reasonable length of shock wave. It is not apparent to the reviewer that this will always be the case.) The drag of the portion of the body forward of the characteristic line is evaluated by application of the momentum theorem around a closed curve consisting of the body, the central streamline, the computed characteristic, and the known bow wave. The drag of the rear portion of the body is determined by a continuation of the characteristics construction in the downstream direction. The general method can also be applied to the determination of the external drag and mass flow of

supersonic inlets when the local flow at the entrance of the inlet is subsonic. No calculated examples are given.

Walter G. Vincenti, USA

948. **André Herpin, A kinetic theory of shock waves** (in French), Rev. Sci. 86, 35–37 (1948).

The author indicates how the flow of a gas through a stationary shock wave can be treated by the methods of statistical mechanics. He illustrates it by the treatment of a monatomic gas. A large number of molecules are considered in a one-dimensional flow. Their velocities have an average forward component plus a random (thermal) component. A general intermolecular force is assumed, and also that the molecules have no internal forms of energy. The author then sets up the equations for the average transport of an arbitrary quantity Q associated with each molecule and depending only on its velocity. By specializing the form of Q he derives statistical equations equivalent to the equations for the conservation of mass, momentum and energy. Taking the flows at $+\infty$ and $-\infty$ to be distinct uniform flows, he derives three shock equations equivalent to the Rankine-Hugoniot equations under the obvious identification of statistical with thermodynamic parameters. By way of addendum the author considers the problem of shock-wave thickness. By assuming this to be of the order of shock-propagation velocity times the relaxation time of the gas, he arrives at a formula for shock thickness which at high shock strengths seems to be more realistic than that derived by the classical means. D. P. Ling, USA

949. **F. I. Frankl', On the formation of shock waves in subsonic flows with local supersonic velocities** (transl. from Russian), Hdqtrs. Air Mat. Comm. Dayton Transl. no. 42, 10 pp. (1949).

This paper argues the plausibility of the fact that a mixed potential flow over a body does not in general exist. The author considers a body which is symmetric both with respect to an axis in the direction of the free stream and to an axis normal to it. The hodograph map of one quadrant of this flow is similar to cases dealt with in Tricomi's boundary-value problem; therefore it is plausible that the same conditions for uniqueness and existence of the solution are valid here. In this specific case it means that the flow is already determined if only part of the supersonic contour of the body is given; the portion of the contour which cannot be prescribed is actually determined by conditions of symmetry. If the given body happens to have such a shape that the portion of the contour which can actually not be prescribed has the shape which is required for the condition of symmetry, then a potential flow is possible. If the prescribed shape deviates from the required contour, then a potential flow will not exist. (Reviewer's remark: The physical phenomena which originate from such a deviation of the contour have been investigated by Busemann and Guderley. See Rev. 2, **1426**). Hideo Yoshihara, USA

950. **Ryuma Kawamura, On the critical phenomena due to shock waves** (in Japanese), Rep. Inst. Sci. Technol. Tokyo 3, 43–48 (Feb. 1949).

A uniform supersonic stream of gas flowing along a stepped wall $OABC$ is deflected by an expansion around a corner A, and then deflected back by an equal angle

$$\theta = \angle OAB - \pi = \pi - \angle ABC$$

at a second corner B, forming an oblique shock wave. If θ is larger than the maximum possible angle of deflection $\theta_{\max}$ pertaining to the shock wave starting at B, the above-mentioned configuration will not take place, but a triangular dead-air region ABB' will be formed, the shock wave starting at a certain point B' on the wall BC. The author assumes that the pressure in the dead-air region is that corresponding to the maximum possible deflection. (But the assumption does not seem to have been established theoretically or experimentally.) The result is applied to the calculation of the base pressure of two-dimensional projectiles. Isao Imai, Japan

951. **M. Z. Krzywobłocki, On certain cases of simple exact solutions of flow equations in a compressible imperfectly viscous fluid with particular conditions,** Math. Mag. 22, 111–123 (1949).

This paper is intended to explore cases of flow of a compressible viscous ideal gas with constant coefficients of viscosity and heat conductivity, where exact solutions, although physically unrealizable, are mathematically possible. These cases are: two-dimensional source, circular vortex and spiral vortex. In all the cases studied, the special form of solutions makes the pressure, density, and temperature, as well as the velocity, equal to zero at infinity. In the case of the vortex, the same assumption necessitates that the temperature be negative in the whole field.

Y. H. Kuo, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 925, 930, 942)

952. **L. Prandtl, Report on investigation of developed turbulence,** Nat. adv. Comm. Aero. tech. Memo no. 1231, 7 pp. (1949).

It is suggested that for turbulent flow in a smooth pipe the shearing stress τ is of the form $\tau = \mp \rho l^2 (\partial u/\partial r)^2$, where u is the velocity and l is the "mean free path" of the eddies. If it is assumed that l is a constant and that τ is proportional to the distance from the center r, we get the velocity profile $u = A - Br^{3/2}$, where A and B are constants. S. Chandrasekhar, USA

953. **H. W. Liepmann, Application of a theorem on the zeros of stochastic functions to turbulence measurements** (in German), Helv. phys. Acta 22, no. 2, 119–126 (1949).

S. O. Rice [Bell System Tech. J. 23, 282–332 (1944); 24, 46–156 (1945); see also M. Kac, Bull. Ann. math. Soc. 49, 314–320 (1943)] has given for the mean number N_0 of zeros of a random stationary function $J(t)$ in an interval of length 1 the following expression:

$$N_0^2 = \overline{J'(t)^2}/\pi^2 \overline{J(t)^2}.$$

This paper is based on the remark that the result of Rice could be applied to any component $u(t)$ of the velocity in a homogeneous isotropic turbulence. The counting of the zeros of $u(t)$ gives then a very simple method for the computation of $\overline{J'(t)^2}$, that is of the well known parameter λ of G. I. Taylor (related to the dissipation of energy by turbulence). Counting the zeros of the transversal velocity (in a low turbulence wind tunnel at C. I. T.) on oscillographic records of an hot-wire anemometer, the author has computed values of λ^2. His main purpose is a comparison with the values given by three other experimental methods (energy spectrum, differentiation of the curve, dissipation of energy). The discrepancy between the four values of λ^2 is of the order of $\pm 10\%$.

Joseph Kampé de Fériet, France

954. **F. Riegels and J. A. Zaat, On the transition of boundary layers into undisturbed flow** (in German), Nachr. Akad. Wiss. Göttingen. Math.-Phys. Kl. 1947, 42–45 (1949).

The authors consider the nonlinear ordinary differential equation describing the similarity solution for the boundary-layer equations when the free-stream velocity varies as a power m of the distance measured from the leading edge along a semi-infinite

flat plate: $f''' + ff'' + 2m(m + 1)^{-1}(1 - f'^2) = 0$. The differential equation is linearized in the region where the boundary-layer velocity approaches that of the free stream by noting that in this region the stream function varies almost linearly with the distance from the plate. The equation becomes

$$f''' + (\eta - \Delta)f'' - 2m(m + 1)^{-1}(f' - 1) = 0,$$

where Δ is a constant, and has solutions for f' of the confluent hypergeometric type. These solutions correspond to the outer solutions of von Kármán and Millikan. When the exponent of the free-stream flow has the value of a half-integer the hypergeometric series becomes a polynomial and the solutions are easily calculable. The solutions so obtained are reasonable even for small distances from the plate, and consequently the arbitrary constant remaining in the solution is obtained through equating the corresponding value of the displacement thickness with the known value of Hartree. The velocity distributions obtained in the region approaching the free-stream condition are in good agreement with Hartree's results, and the authors derive a Lagrangian interpolation formula which they use to obtain solutions for values of m other than those explicitly computed.
Courtesy of Mathematical Reviews F. E. Marble, USA

955. Hans W. Liepmann and Gertrude H. Fila, Investigations of effects of surface temperature and single roughness elements on boundary-layer transition, Nat. adv. Comm. Aero. Rep. no. 890, 12 pp. (1947).

The laminar boundary layer and the position of the transition point were investigated on a heated flat plate. It was found that the Reynolds number of transition decreases as the temperature of the plate is increased. It is shown from simple qualitative analytical considerations that the effect of variable viscosity in the boundary layer due to the temperature difference produces a velocity profile with an inflection point if the wall temperature is higher than the free-stream temperature. This profile is confirmed by measurements. Furthermore, it is confirmed that, even with large deviation from the Blasius condition, the velocity and temperature profiles are very nearly identical, as predictable theoretically for a Prandtl number of the order of 1.0. The instability of the inflection-point profiles is discussed.

Studies of the flow in the wake of large, two-dimensional roughness elements are presented. It is shown that a boundary layer can separate and reattach itself to the wall without transition taking place. R. C. Binder, USA

956. E. B. Klunker and H. Reese Ivey, An analysis of supersonic aerodynamic heating with continuous fluid injection, Nat. adv. Comm. Aero. tech. Note no. 1987, 31 pp. (1949).

On the basis of established relations for laminar film cooling an analysis is given of steady-state supersonic aerodynamic heating with fluid injection. Charts are given showing the extent of cooling at various altitudes and Mach numbers. A brief stability consideration shows that the reduced surface temperatures assist in maintaining stability for a wide range of conditions. The skin-friction drag coefficient is calculated approximately for various amounts of cooling. Newman A. Hall, USA

957. L. E. Kalikhman, Gas dynamic theory of heat, Hdqtrs. Air Mat. Comm. Dayton, Transl. no. A9T, 51 pp. (Jan. 1949).

The author develops the theory of the compressible laminar and turbulent boundary layer for fluids of different Prandtl number σ and where the dynamic viscosity is proportional to T^n. Separate solutions are given for the two-dimensional and the axisymmetric flow.

By means of the transformation $\eta = {}_0\int^y \rho/\rho_0\, dy$, where ρ_0 is the free-stream stagnation value of the density, the equations of momentum and energy are simplified and can be solved by the standard methods used in incompressible flow. Similar transformations have been used by L. Howarth [Proc. roy. Soc. London Ser. A, 194 (1948); Rev. **2, 357**] and A. A. Dorodnitsín [Prikl. Mat. Mekh. 6, 449–485 (1942)].

When $\sigma = 1$, the laminar solution is obtained by the von Kármán-Pohlhausen method for incompressible flow. An approximate method is developed to construct integral forms for both the momentum and temperature thicknesses, and from these values the local skin friction and heat transfer can be obtained.

The turbulent solution is obtained by use of the Prandtl-von Kármán method for incompressible flow, modified to include compressibility effects, although density fluctuations are ignored. Finally the Squire-Young method for the calculation of profile drag of an airfoil is modified to include the effect of compressibility.

Results of calculations of the heat transfer around a wedge section at free stream Mach numbers of 2 and 6 are given in graphical form. It is shown that in this example the Nusselt number for the laminar layer is nearly independent of Mach number, but for the turbulent layer the Nusselt number decreases with increase of Mach number.

G. M. Lilley, England

Aerodynamics of Flight; Wind Forces

(See also Revs. 943, 946)

958. John H. Quinn, Jr., Summary of drag characteristics of practical-construction wing sections, Nat. adv. Comm. Rep no. 910, 19 pp. (1948, publ. 1949).

Results of a number of wind-tunnel tests have been collected and analyzed to determine the effects of surface roughness and waviness due to construction irregularities, buckling of skin due to compressive load, and deicers upon drag characteristics. The improvement which could be obtained by combinations of surface finishing and fairing the irregularities is also evaluated and discussed. John E. Goldberg, USA

959. M. Kohler and W. Mautz, Pressure-distribution measurements on the tail surfaces of a rotating model of the design BFW-M31, Nat. adv. Comm. Aero. tech. Memo. no. 1220, 38 pp. (1949).

This investigation was initiated as a means of determining the load distribution on the tail surfaces of an airplane in attitudes comparable to those found in a spin. A model-support system was developed consisting of a vertical shaft about which the model could be driven in rotation. Provision was made for variations in the angle of attack setting from 0 deg to 90 deg and in the angle of yaw from -10 deg to $+10$ deg. Adjustments of elevators and rudder were also possible. Pressure orifices over the tail surfaces were connected to separate manometers by means of a system of so-called Fuhrmann-type gaskets, developed originally for pressure-distribution tests on the blades of a rotating propeller.

A limited amount of test data are presented for only one angle of attack of 60 deg. The results show the existence of an unsymmetric distribution on the horizontal tail surfaces tending to maintain the rotation, as would be the case in autorotation. The effects of wing and fuselage shielding are indicated. The program apparently was the initial phase of a much more extensive schedule of tests. Thus the report is of primary interest because of its description of the technique employed and the general nature of the results which could be obtained. M. J. Thompson, USA

960. **William Letko and David Feigenbaum, Wind-tunnel investigation of split trailing-edge lift and trim flaps on a tapered wing with 23° sweepback,** Nat. adv. Comm. Aero. tech. Note no. 1352, 72 pp. (July 1947).

Results of force tests and pressure-distribution measurements are presented from a wind-tunnel investigation to determine the effects of size and hinge location of lift and trim flaps on the lift and pitching-moment characteristics of a semispan tapered wing with 23-deg sweepback of the quarter-chord line. The flaps tested were split flaps with chords of 10, 20, 30, and 40% of the wing chord. The spans of the lift flaps were 20, 40, 60, 80, and 100% of the wing span; the spans of the trim flaps were 10, 20, 40, and 60% of the wing span. The flaps were tested with the hinge axes at several different chordwise locations.

The static longitudinal stability of the sweptback wing, as indicated by the slope of the curves of pitching-moment coefficient against lift coefficient, was increased when the lift flaps were deflected, especially for the larger flaps.

Increments in maximum lift coefficient of the order of 0.4 were produced in some configurations by self-trimming lift flaps, that is, lift flaps that produced no increment in pitching moment about the aerodynamic center. By the use of trim flaps to counteract the pitching moments produced by the lift flaps, increments in maximum lift coefficient of the order of 0.5 might be attained. The chord of the trim flap used had a negligible effect on the net lift coefficients attainable, although use of a large-chord trim flap meant that a smaller span was required. Using a trim flap with the hinge axis moved back to the trailing edge, however, allowed slightly greater lift increments to be attained. The increments in trimmed lift coefficient produced by the lift flap increased with flap chord and reached a maximum value for all flap chords at a flap span of about 50% of the wing span. Moving the hinge axis of the lift flaps forward increased the lift-coefficient increment attainable at a 10-deg angle of attack with self-trimming flaps. The greatest increment in maximum lift coefficient attainable with self-trimming flaps occurred, however, when the flap hinge axis was on the 70%-chord line. A comparison of the results with analytical results showed, in general, reasonably good agreement. Authors' summary

961. **John P. Campbell and Alex Goodman, A semiempirical method for estimating the rolling moment due to yawing of airplanes,** Nat. adv. Comm. Aero. tech. Note no. 1984, 30 pp. (1949).

This report presents a method by which the difference between experimental and theoretical values of $C_{l\beta}$ can be used to correct theoretical values of the rotary derivative, C_{l_r}. Experimental values of C_{l_r} for 22 different wings and 8 complete models are used to show the improvement this method has over existing theoretical methods for evaluating C_{l_r}. Harvard Lomax, USA

962. **Marion O. McKinney, Jr., Analysis of means of improving the uncontrolled lateral motions of personal airplanes,** Nat. adv. Comm. Aero. tech. Note no. 1997, 14 pp. (1949).

Rolling and yawing motions of a hypothetical personal plane following various disturbances are calculated using the equations given in Report no. 589 from same source. It is shown that an increase in spiral stability can be obtained by increasing tail length or by increasing vertical tail area and dihedral angle simultaneously without adversely affecting flying qualities.

Dana Young, USA

963. **Harry Greenberg, Frequency-response method for determination of dynamic stability characteristics of airplanes with automatic controls,** Nat. adv. Comm. Aero. Rep. no. 882, 9 pp. (1947).

Examples are given to illustrate the application of the frequency-response method in determining oscillation characteristics of airplanes with an automatic pilot. Dana Young, USA

964. **Sidney M. Harmon, Theoretical relations between the stability derivatives of a wing in direct and in reverse supersonic flow,** Nat. adv. Comm. Aero. tech. Note no. 1943, 47 pp. (1949).

The computation of aerodynamic characteristics or stability derivatives for wings is rather tedious even in cases of simple geometric plan forms. It is therefore useful to establish relations between these coefficients for related geometrical forms. It is known that for some wings some characteristics do not change their value if the direction of flow is reversed. The author investigated completely the influence of flow reversal on all coefficients needed for studies of symmetric flight and for the rolling of an airplane. The wings investigated are thin flat plates and have a supersonic leading edge, a supersonic trailing edge, and either supersonic side edges or subsonic nonreentrant side edges which are noninteracting. Irmgard Flügge-Lotz, USA

965. **Arthur W. Carter, Recent N.A.C.A. research on high length-beam ratio hulls,** J. aero. Sci. 16, 167–183, (Mar. 1949).

A series of related seaplane hulls having a wide variation in length-beam ratio have been investigated by the Langley Laboratory of the NACA. An increase in hull length-beam ratio from 6 to 15, holding the (length)^2-beam product constant, reduced the aerodynamic drag without appreciably affecting the hydrodynamic qualities in smooth water, reduced the vertical accelerations and motions during landings in rough water, and reduced the structural weight required for a given load factor.

Extreme warping of the forebody planing bottom of length-beam ratio 15 resulted in large reductions in the maximum vertical and angular accelerations during landings in rough water, and considerably improved the spray at low speeds. Warping the forebody and extending the afterbody resulted in further reductions in the maximum accelerations in rough-water operation.

The high length-beam ratio hull, which has been shown to have acceptable hydrodynamic characteristics, is therefore of particular interest in the design of high-performance water-based aircraft, inasmuch as appreciable reductions in aerodynamic drag and hull weight may be obtained. Such high length-beam ratios combined with long afterbodies and warped forebodies are markedly superior to conventional length-beam ratios for rough-water operation.

Outstanding progress has been made in recent years in the field of seaplane hydrodynamics which has done much to bring the faltering seaplane up to modern standards of aerodynamic performance and design efficiency. Ernest G. Stout, USA

966. **Walter J. Kapryan, Effect of forebody warp and increase in afterbody length on the hydrodynamic qualities of a flying-boat hull of high length-beam ratio,** Nat. adv. Comm. Aero. tech. Note no. 1980, 36 pp. (1949).

In an attempt to achieve improved hydrodynamic qualities for flying boats, an investigation was made to determine the combine effect of a warped forebody and extended afterbody on the hydrodynamic characteristics of a hull having a basic length-beam ratio of 15. Each of these modifications has been found to result in improvements in hydrodynamic qualities, and their combined effect is therefore of interest.

The stable range of trims available for take-off was increased by warping the forebody and extending the afterbody. For take-off at constant elevator deflection, the range of center-of-gravity positions for satisfactory take-off stability was increased. The smooth-water landing stability remained satisfactory. Spray

characteristics were improved, no spray entering the propellers or striking the flaps at the design gross load. Definite improvements in rough-water landing behavior were obtained with the combination of warped forebody and extended afterbody. For landings in waves 4 ft high, the impact accelerations were reduced more than 50%. The maximum amplitudes of oscillation in trim and rise occurring during the landing run-out were also substantially decreased. In general, the changes in hydrodynamic characteristics effected by combining these modifications were in the direction that would be predicted on the basis of results obtained for the individual modifications. Although the effects were not directly additive, they were cumulative, and changes in opposite directions were compensating.

Ernest G. Stout, USA

Aeroelasticity (Flutter, Divergence, etc.)

967. George E. Castile and Robert W. Herr, Some effects of density and Mach number on the flutter speed of two uniform wings, Nat. adv. Comm. Aero. tech. Note no. 1989, 20 pp. (1949).

Cantilever wing models of NACA 16-010 section were operated at zero attack angle and sweepback angles of 0 deg and 15 deg, in the Langley flutter tunnel. Flutter velocities and frequencies were observed over the following ranges: Mach number, 0.22 to 0.82; density of medium, 0.00029 to 0.0085 slug/cu ft; mass ratio, 0.00375 to 0.11; Reynolds number, (0.27 to 3.00) $\times$ 10^6. Observed flutter velocities were multiplied by air-force correction functions involving Mach number and circulation; a nearly linear relation then obtained with the reduced density (reciprocal of the mass ratio). This indicates that the dynamic pressure at flutter is nearly constant over a wide range of densities. Effects attributable to Reynolds numbers above 0.4 $\times$ 10^6 and to moderate tunnel-wall interference were found inappreciable in the range of variables investigated. Vincent Salmon, USA

968. John E. Baker and Russell S. Paulnock, Experimental investigation of flutter of a propeller with Clark Y section operating zero forward velocity at positive and negative blade-angle settings, Nat. adv. Comm. Aero. tech. Note no. 1966, 13 pp. (1949).

This is a brief account of tests conducted on a small, one-blade wooden propeller. The blade setting at 0.8 radius was varied from -25 deg to $+25$ deg, and the mode of flutter was found to undergo a transition from classical type (torsion-bending) at small positive blade angles to the stalling form (predominantly torsional) at large positive and negative angles. The method of Theodorsen and Regier [same source, ACR L5F30) was employed to calculate a critical speed for classical flutter. Observed tip speeds at the onset of flutter range from 18% to 63% of the theoretical flutter speed. M. J. Turner, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 858, 923, 940, 968, 1010)

969. Herbert S. Ribner, A transonic propeller of triangular plan form, Nat. adv. Comm. Aero. tech. Note no. 1303, 41 pp. (May 1947).

A propeller for transonic flight is proposed which avoids the high bending stresses caused by centrifugal forces limiting the amount of sweep that can be used. It permits designs for Mach numbers previously considered structurally impracticable. It is shaped from an isosceles triangle twisted into a screw surface about its axis. The purpose is to attain the wave-drag reduction associated with large sweepback in a structurally practicable configuration. The theory for such a propeller is based in R. T. Jones' work in low-aspect ratio pointed wings [same source, no. 1032 (1946)] and on H. E. Puckett's work on thin airfoils [J. aero. Sci. 13, 475–484 (1946)].

Calculations taking account of wave and skin-friction drag indicate a net efficiency of the order of 80% at Mach number 1.1. A 12-ft propeller is estimated to be able to absorb 18,500-brake hp at 840 mph at sea level. No experimental tests are discussed.

M. F. Dowell, USA

970. E. Maillet, Development of centrifugal compressors (in French), Rech. aéro. Paris no. 2, 27–34 (Mar. 1948).

The author reviews the development of turbocompressors working in connection with turbomachines of single circuit used in aviation.

He discusses especially the centrifugal type with and without front shrouds and without any shrouds. Radial blades and both backward and forward curved blades are compared. The reviewer, although generally agreeing that forward curved blades are not as good as radial or backward curved ones, cannot accept the particular explanation the author gives for this fact. The possibility of obtaining higher stage-pressure ratios by increasing the peripheral speed is studied. Recompression by shock in the diffusor is discussed. Comparison is made between axial-, mixed- and radial-flow turbocompressors with respect to stage-pressure ratio, efficiency, space and weight needed, and durability. The article gives German, French, English and American references.

Wilhelm Spannhake, USA

971. Sugao Sugawara, Takashi Satō and Harumi Uchiyama, The effect of steam-turbine-blade pitch on its velocity coefficient, 1st report (in Japanese), Trans. Soc. mech. Engrs. Japan 13, no. 45, 186–197 (1947).

Wind-tunnel tests of a cascade of thickened blades with equal inlet and exit angle are reported. Four kinds of blades with angles of 20, 30, 40, 50 deg were used, and for each kind five blades were arranged in cascade. Only the case of shockless entrance is considered. Distributions of pressure and velocity at the inlet and exit sections are measured with pitot and static tubes. Values of velocity coefficient ψ ($= w_2/w_{2a}$, where w_2 is the mean velocity at the exit, and w_{2a} is the theoretical exit velocity) were determined for various pitches of the blades. The authors give a formula for the maximum-ψ pitch: $s = 1.03\, r/(2 \sin \beta)$, where r is the radius of curvature of the blade, and β is the blade angle. This formula is to be compared with Briling's formula for sheet-metal blades, $s = r/(2 \sin \beta)$. Humio Tamaki, Japan

972. Boleslaw Szczeniowski, Study of efficiency and thrust of conventional turbo-jet propulsor, Rev. trim. Canad. 35, 256–280 (1949).

The author presents an analytical investigation of the effect of turbine and compressor adiabatic efficiencies on the over-all thermal efficiency of a turbojet engine. Higher turbine and compressor adiabatic efficiencies permit higher optimum-pressure ratios across the compressor, and hence higher thermal efficiencies. The over-all gain seems to be considerable for relatively small changes in component efficiencies. No experimental data are presented. Joseph F. Manildi, USA

973. John C. Sanders and Virginia L. Brightwell, Analysis of ejector thrust by integration of calculated surface pressures, Nat. adv. Comm. Aero. tech. Note no. 1958, 36 pp. (1949).

An analysis of the thrust of air ejectors is carried out by calculating the pressure forces on the walls of the ejector. The one-dimensional approximation is used for determining the pressures.

General equations are established for incompressible flow, whereas the case of compressible flow is only illustrated by means of some numerical examples. The decrease in thrust at high velocities of the secondary air stream and the influence of the density ratio of the primary and secondary streams entering the mixing zone are well illustrated by means of diagrams. It is shown for the incompressible case that the thrust is essentially created by pressure forces on the walls of the converging secondary passage preceding the mixing zone. In the case of compressible flow the calculated examples show a decrease in thrust as compared with the incompressible case. E. Haenni, Switzerland

Flow and Flight Test Techniques

(See also Revs. 910, 959)

974. **G. Drougge, A method for the continuous variation of the Mach number in a supersonic wind tunnel and some experimental results obtained at low supersonic speed,** Flygtekn. Försöksanst. Medd. Rep. no. 29, 14 pp. (1949).

A description is given of a method of obtaining experimental results over a wide range of Mach numbers in a fixed supersonic wind-tunnel nozzle. The technique involves the placing of models in the flow field of a movable two-dimensional wedge located in the wind-tunnel test section. The Mach number at the model position is varied by changing the angle of attack of the movable wedge.

In a nozzle with a free-stream Mach number of 1.39, lift and drag data were obtained for a two-dimensional wing of biconvex section, thickness ratio 0.08, over the Mach number range from 1.44 to 1.12. With a small conical model, semiangle 10 deg, Mach numbers from 1.45 to 1.04 were obtained.

The flexibility and simplicity of the method make it immediately useful for demonstration purposes and for exploratory research. More detailed investigations are required before it can be regarded as a dependable source of quantitative information.

Alvin R. Eaton, Jr., USA

975. **E. Petersohn, On maximum model size for wind tunnel investigations at high subsonic Mach numbers,** Flygtekn. Försöksanst. Medd. no. 23, 1–12 (1948).

By means of two-dimensional measurements in a closed wind tunnel with a circular test section, an experimental relation has been established between the relative model size and the maximum velocity attainable without serious interference of the tunnel walls.

Pressure-distribution measurements on three NACA 0012 wings with chords of 140, 240 and 412 mm showed a systematic influence only at the highest velocities and at the rearmost measuring points. A rapid pressure drop apparently occurred at lower velocities with larger models. For the NACA 0012 wing of 140-mm chord, equipped with end pieces, the effect was studied of a spanwise varying construction in the tunnel midsection.

Velocity distribution was investigated (also for NACA 0016 and 0008 profiles of 240-mm chord length) with the result that at a certain Mach number, sharply defined by the model size, the velocities increased strongly.

Finally, a comparison was made with the constriction at which adiabatic flow would become sonic at the minimum tunnel section with the model in place. M. F. Dowell, USA

976. **W. James Orlin, Norman J. Lindner, and Jack G. Bitterly, Application of the analogy between water flow with a free surface and two-dimensional compressible gas flow,** Nat. adv. Comm. Aero. Rep. no. 875, 18 pp. (1947, publ. 1949).

The theory of the analogy between water flow with a free surface and the two-dimensional motion of a compressible gas is briefly reviewed and evaluated. The evaluation is based primarily upon a qualitative comparison of data for flow around cylinders obtained in the water channel and in the high-speed wind-tunnel, and includes a thorough discussion of theoretical restrictions. The hydraulic apparatus and measuring equipment are discussed in detail and, on the basis of observed results, general recommendations for such apparatus are made.

John E. Goldberg, USA

977. **Arturo Dolcetta, Experimental equipment for the solution of hydrodynamic problems by the method of electric analogy** (in French), Energia Elett. 26, 461–468 (Aug. 1948).

The author first reviews the theoretical principles of the electrical analogy of hydrodynamics. Next, he describes the equipment used by the Hydraulic Institute of the Milan Polytechnic for experimental study of seepage. Finally, he presents some experimental results in a theoretically solved problem, thus demonstrating the quality of the approximation and the possibility of further improvement. L. Escande, France

Thermodynamics

(See also Revs. 909, 919, 948, 972, 984)

978. **O. K. Rice, The thermodynamics of liquid helium on the basis of the two-fluid theory,** Phys. Rev. 76, no. 11, 1701–1707 (1949).

The thermodynamics of liquid helium is developed in the light of the two-fluid theory which states that below the λ point, liquid helium consists of a superfluid and a normal phase. The thermodynamic consequences of assuming these phases to be in equilibrium are developed. The specific heat just above the λ point is considered on the assumption that in this region the superfluid is beginning to appear in the form of small globules containing around 60 to 80 helium atoms. The entropy of mixing of the two fluids below the λ point is established, which result suggests that in this region the superfluid forms a "partially condensed system." The nature of the second-order transition at the λ point, and the change of the λ point with pressure and He_3 concentration are considered.

After the author's summary by Newman A. Hall, USA

979. **Rolf Haase, Some general relations between the properties of multicomponent systems** (in German), Z. Naturforsch. B 3a, 285–290 (May 1948).

A relationship between partial molar and mean molar quantities is derived from thermodynamics. The geometric procedure for obtaining the partial molar quantities for binary and ternary mixtures follows from this, as does a simplification in the method of obtaining partial molar properties of systems of many components. The relationships for the coexistence of two phases are obtained in such a form that the basis of the geometrical method arises as a special case of the general equation for many components. Several applications are discussed.

Keith J. Laidler, USA

980. **A. Cizek, Thermoelastic effects** (in Czech), Publ. Fac. Sci. Univ. Masaryk no. 289, 38 pp. (1947).

The article describes several testing methods for verification of the second law of thermodynamics. A brief review is given of thermoelastic phenomena during tension, torsion and bending of solids, liquids and gases. The following apparatus are described: for pulling a wire and rubber, for straining a wire in tension, for

bending a celluloid strip, for compressing paraffin, methanol, gases and air. L. M. Tichvinsky, USA

981. G. S. Rushbrooke, Introduction to statistical mechanics, Oxford, Clarendon Press, 1949, 334 pp., 48 figs. Cloth, 5.7 × 9 in., $5.50.

This book gives a clear, authoritative and accurate account of the fundamentals of statistical mechanics. It is written primarily for students of physical chemistry, and therefore assumes only a limited mathematical knowledge; on account of the originality of its approach, however, it should also be of great value to workers in physics and mechanics. The chief emphasis is placed on fundamental principles and methods, although a number of applications to experimental problems are made in some detail. A clear explanation is given of the three basic methods of statistical mechanics, namely, the number of complexions, the partition function and the grand partition function. The distinction between the methods of statistical mechanics and thermodynamics is preserved, and some previous knowledge of the latter is assumed; the final chapter discusses briefly the connection between the two treatments.

A valuable feature of the book is that it gives a particularly clear account of certain physical phenomena; e.g., specific heats, molecular spectra, hindered rotation, an understanding of which is important for a proper appreciation of the applications of statistical mechanics. The first five chapters (80 pages) are mainly concerned with fundamental principles, with, however, constant reference to physical concepts. Perfect gases are then considered; the various types of partition functions are evaluated and applied to problems of specific heats of diatomic and polyatomic molecules, the equilibrium between ortho- and parahydrogen, hindered rotation in molecules, the third law of thermodynamics, etc. The law of mass action and two-phase assemblies are then discussed. The idea of the partition function for an assembly is then introduced, and applied to imperfect gases. The method of the grand partition function is then developed and applied to condensed systems, including nonperfect solutions.

Keith J. Laidler, USA

982. A. Münster, On the statistical mechanics of regular and irregular solutions (in German), Z. Naturforsch. A 3a, 158–172 (Mar. 1948).

A method is developed which, with some assumptions, makes it possible to carry out an approximate calculation of the partition function of liquid mixtures of arbitrarily many components of arbitrary molecular size and structure. It is assumed that: (a) one component, the solvent, is present in more than 50% of the total mass; (b) the molecules of the solvent are not larger than those of the remaining components and are approximately spherical; (c) only short-range intermolecular forces are present. The result is applied to the problem of the strictly regular binary solutions as well as that of mono- and polydispersed solutions of high linear polymers. In the last case the thermodynamic properties in general depend on the partition function of the degree of polymerization. Serge Gratch, USA

983. H. Tompa, Statistical thermodynamics of mixtures of normal paraffins, Trans. Faraday Soc. 65, 101–109 (Jan. 1949).

The treatment of the statistical thermodynamics of mixtures developed by E. A. Guggenheim is extended to mixtures of molecules of different length but built of identical terminal segments and identical internal segments. The resulting formulas are applied to Brønsted and Koefoed's measurements of the activities in mixtures of normal paraffins, and the heat of mixing is calculated. Edgar F. Westrum, Jr., USA

Heat Transfer; Diffusion

(See also Revs. 956, 957, 1007)

984. J. Danze, Research on industrial drying (in French), Hommage Faculty appl. Sci. Univ. Liége, 72–82 (1947).

The author has investigated both theoretically and experimentally the mechanism of the drying of a hygroscopic material. The reasons for the changes in the rates of evaporation during the different stages of the drying cycle are fully described.

Calculations of the diffusion of water during the "constant rate period" from the interior to the surface of a flat sheet of material are given, but the results are not checked by experiment. A difficulty in applying the theory is that the coefficient of diffusion between the liquid and material is unknown. Similarly some ideas are quoted for a theoretical attack on the problem of diffusion during the "unsaturated surface periods," but the analysis is not completed.

A number of experiments are reported on the drying, under conditions of forced convection, of sheets of cardboard, plaster, celotex and wood. The results are plotted in the form rate of evaporation, at different wind speeds, against mean moisture content of the material. An empirical formula, satisfied by the experimental results from all the materials tested, is derived for the constant rate period. G. M. Lilley, England

985. W. H. Klein, Calculation of solar radiation and the solar heat load on man, J. Met. 5, 119–129 (Aug. 1948).

The author develops a method for computing the intensities of direct, diffuse and total solar radiation impinging on a horizontal surface of the earth. Sky radiation, terrain reflection, and depletion by dry air, water vapor, dust and clouds are considered. Methods for the determination of all these factors are given. The calculation is the extended aloft, and an equation derived expressing the atmospheric-transmission coefficient with cloudless sky, at any altitude between 300 and 8000 meters, as a linear function of solar zenith distance and the logarithm of the elevation.

The human body receives solar heat from direct, diffuse and reflected radiation. Of this total, the portion absorbed by the skin constitutes the solar heat load on man. This can be evaluated and conveniently expressed as an equivalent increment of the temperature, whose addition to the actual temperature produces the "environmental temperature." M. F. Dowell, USA

Acoustics

986. Harold Levine and Julian Schwinger, On the radiation of sound from an unflanged circular pipe, Phys. Rev. (2) 73, 383–406 (1948).

The authors solve the problem of sound radiation from a semi-infinite unflanged circular pipe, assuming axially symmmetric excitation. The solution found is valid in the wave-length range of dominant mode propagation in the pipe. The problem is formulated as an integral equation of the Wiener-Hopf type for the radial derivative of the velocity potential. The solution of this integral equation is found by the methods of Wiener and Hopf employing the Fourier transform in the complex domain. The factoring of the transform equation is accomplished with the aid of Cauchy's integral theorem. The factors which contain terms given by integrals in the complex domain are reduced to real integrals and then evaluated numerically. From this the reflection coefficient for the velocity potential within the pipe and the power gain function are evaluated numerically.

A. E. Heins, USA

987. **R. Kling and J. Christophe, Sound velocity in hydrocarbons and its relations to molecular structure** (in French), Rech. aéro. Paris no. 7, 59–64 (Jan.-Feb. 1949).

This paper is a short discussion of the status of sound velocity and absorption studies in liquids. No original experiments are presented or theories proposed. The paper begins with an enumeration of the main experimental methods that have been employed successfully for studying the propagation of sound in liquids. This is followed by a short discussion of Kneser's theory of dispersion in gases and its extension to liquids. Reported data on dispersion in liquids, however, are very confusing.

It appears to be generally true that appreciable dispersion is always accompanied by large absorption. Mention is made of the theories of absorption proposed by Stokes, Kirchoff, R. Lucas, Kneser, Clayes, Errera, Sack, and Burton. These theories generally predict effects much smaller than are found experimentally.

Some success has been obtained in relating the velocity sound to the molecular structure of homologous hydrocarbons, particularly through the use of an empirical expression due to Rama Rao. Expressions have been proposed by Rao, Utter and Kling, and Burton for predicting the velocity in mixtures. As for absorption in this case, theories have been proposed by Herzfeld, Spakovsky, and Parshad. It appears that association must play an important role in absorption considerations. Horace M. Trent, USA

988. **J. Hartmann and T. Mortensen, A comparison of the Rayleigh disk and the acoustic radiometer methods for the measurement of sound-wave energy,** IngenVidensk. Skr. 1948, no. 2, 3–65.

Elaborate experimental investigations of the radiation pressures produced by sound waves impinging on thin plates (radiometer), and the torques produced on thin strips (Rayleigh disks) were carried out. The object of the experiments was to determine the validity of the usual formulas for deducing sound intensity from radiometer and disk measurements. The authors conclude that the pressure on an acoustic radiometer in air is twice the energy density in the sound wave. The measurement of these second-order effects due to sound waves is very difficult, and the results are of general interest. However, it should be pointed out that the Rayleigh disk and acoustic radiometer have been superseded, as intensity measuring devices, during the last 20 years by microphones and associated electronic apparatus.

Richard K. Cook, USA

989. **Juichi Igarashi, On the measurement of the sound-absorption coefficient** (in Japanese), Rep. Inst. Sci. Technol. Tokyo 2, 162–164 (Oct.-Nov. 1948).

A new method of measuring the sound-absorption coefficient of various materials is presented, utilizing the fact that the decay of eigenvibrations of air columns in pipes is perfectly logarithmic. The method is especially suitable for the case of a small absorption coefficient for which the usual "standing wave" method fails. The test material is attached to the inside of one of the closed ends of a brass pipe, and a sound wave is introduced through a small hole at the center of the opposite end. A small crystal microphone is used for measuring the sound pressure. The decay of eigenvibrations in the pipe is measured by comparing it with the voltage decay of R-C circuit. The electrical circuit for this comparison is similar to that of Olson and Kreuzer [J. acoust. Soc. Amer. 2, 78–82 (1930)]. The absorption coefficient of the test material is determined from the measured values of decay time for the pipe with and without the material. A comparative measurement by the standing-wave method is carried out for materials with fairly high absorption coefficient. Agreement of the results is satisfactory. The author remarks that the present method is not accurate when the absorption coefficient is too high.

Humio Tamaki, Japan

Ballistics, Detonics (Explosions)

990. **M. Garnier, Ballistics of a gyro-projectile, II** (in French), Mémor. Artill. fr. 21, no. 3, 615–681 (1947).

This paper treats the problem of the motion of a spinning shell through moving air. It is a sequel to an earlier paper (1947) by the same author on the ballistics of a spinning shell moving through still air. After deriving the differential equation of the motion through moving air, the author demonstrates its application by a step-by-step calculation of a typical trajectory in a wind, utilizing aerosonde wind data. This numerical example is given in very complete detail. R. Smelt, USA

991. **P. Blanc, Principles of exterior ballistics of propulsion engines of the type of V-2 rocket** (in French), Mémor. Artill. fr. 23, no. 88, 311–385 (1949).

The paper's aerodynamics is dubious and out of date, the kinematics conventional, and the remainder essentially known.

A. W. Wundheiler, USA

Soil Mechanics, Seepage

(See also Revs. 851, 852, 887, 888, 924, 977)

992. **Leonard P. Capper and W. Fisher Cassie, The mechanics of engineering soils,** New York, McGraw-Hill Book Company, Inc., 1949, 270 pp., 122 figs. Cloth, 5.6 × 9 in., $4.

With the object of unifying scattered information, this book explains the basic principles of soil mechanics, describes the more usual tests, and introduces the reader to some of its practical applications. It presents a rather complete and clear framework of the field of soil mechanics, but, because it touches on so many subjects, some are merely outlined.

After an introductory history, the soil-test equipment and procedures are described along with formulas required for calculating the test results as usually reported. Subsequent chapters consider earth pressures on walls and bulkheads; slope analysis by slices and by stability numbers; elastic-stress distribution and bearing capacity below footings; consolidation settlement illustrated by complete example for magnitude and rate; reasonable use of piles and load tests; compaction, base stabilization and thickness design for roads and runways; permanent drainage installations and temporary control of water for excavations; aims and procedures for exploration.

While the importance of water in the mechanics of soils is often mentioned, its quantitative effects are not intensely developed. The limitations of calculations for idealized conditions are expressed but are not generally illustrated by examples.

The text is progressive in showing how settlements may be calculated directly from strains without the introduction of voids ratios. The relaxation method of flow-net construction is explained. On the other hand, water films are overworked as in an insupportable explanation of bulking of sand.

References to standard practice are generally British. This book could serve as the basis for a first course in soil mechanics, although there are no problems included.

Edward S. Barber, USA

993. **József Jáky, Validity of Coulomb's law of stability** (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 87–90 (1948).

Through considerations of the geometry of the circle of stress, of Cauchy's equations and of general stability, the author develops an equation for the envelope of the critical circles of stress (failure curve).

It is shown that, in the circle tangent to a point (t, n) of the curve, the stresses are all combined functions of the slope angle of the curve and the slope angle of the sliding surface considered. This latter angle may be considered either constant or a function of the slope of the curve, under arbitrary conditions, from which the unit weight may also be considered a function of the slope of the curve of failure.

Differential equations may be established and solved for cases like the stress conditions in the shearing test where, the unit weight being negligible, the law $t = c + n \tan \phi$ is shown to hold, although another solution is shown to be applicable in the range of tension. The case for heavy earth masses is solved generally, and it is proved that the stresses can be computed as soon as the shape of the sliding surfaces, and the variation of the unit weight with depth are determined. Robert Quintal, Canada

994. József Jáky, State of stress at great depth (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 90–93 (1948).

The aim of this paper is to test the validity of two laws, already well proven, for pressures of the order of 10 kg/cm², when the pressures are greatly increased. One is Terzaghi's law that the voids ratio depends on the logarithm of the pressure, and the other is Coulomb's law that the shearing strength is equal to a constant plus a linear function of the normal stress. The solution is partly experimental and partly theoretical.

The compression-test results were plotted both on semilog and full logarithmic graphs and show distinctly that for the fine sand used in the test, Terzaghi's law is valid up to about 4 kg/cm², after which a power-parabolic law should be used. As a consequence, the author notes that the geostatic pressure is not a linear function of the depth but may give values 10% too small.

High-pressure shearing tests carried out by the Geotechnical Committee for Government Railways of Japan, and the plotting of some Massachusetts Institute of Technology tests on full logarithmic scales show that a power-parabolic law should be used to express the shearing strength with respect to the normal stress. Theoretical considerations of the coefficients of the power parabola are given, showing that in certain cases Coulomb's law is legitimate but that it is only a special case of a more general theory. Robert Quintal, Canada

995. S. G. Bergstrom and S. Linderholm, A test with dynamical soil examination (in Swedish), Tekn. Tidskr. 77, no. 39, 705–709 (1947).

In this test the authors apply a shaking moment to the soil by means of a mechanical shaker for the purpose of obtaining information regarding the dynamic constants of the soil material. Static tests were also made but were not considered by the author to be as satisfactory as the dynamic ones.

N. O. Myklestad, USA

996. A. Mortensen, Predetermination of the resistance of a pile on basis of field shear and loading test values, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 180–185 (1948).

The point resistance and skin friction of six full-scale piles, driven some 40 ft into stiff clay in the harbor of Aalborg, Denmark, were determined by means of successive loading and pulling tests on individual piles in the field. The values thus obtained were correlated with the friction and the bearing values obtained during the extraction of some 280 clay samples from bore holes in the harbor area. The special type of loading-test sampler and the techniques employed for the purpose are described in another paper by the same author [same source, 1, 244–248]. Further correlations were made between the above values, the driving records of the full-scale piles, and laboratory cone-penetration tests on undisturbed and disturbed samples of the clay. It is claimed that a satisfactory procedure has been developed for predetermining, in advance of the construction, the bearing capacity of piles in similar soil formations by means of the sampling and testing techniques described by the paper.

Gregory P. Tschebotarioff, USA

997. E. de Beer and M. Wallays, Limitation of the validity of application of the formulas from Prandtl-Buisman and from Anderson for the ultimate bearing capacity of the soil underneath footings, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 63–68 (1948).

A comparative analysis of bearing capacity formulas for wall-type footings is presented. The results of the formula of Prandtl-Buisman are compared with those of the formula of Anderson.

The first of these formulas is based upon the assumption that the surface of failure is a composite surface, the cross section of which is made up of a segment of a logarithmic spiral with linear extensions. The latter formula is based upon the assumption that the surface of sliding is a composite of two cylindrical segments. In particular, the influence of the ratio of the width to the depth of the footing is investigated for the case of a cohesionless material. The analysis indicates the effect that the shape of the assumed sliding surface has upon the computed bearing values.

For small values of the foundation depth-footing-width ratio, Anderson's formula yields smaller values than the Prandtl-Buisman formula; for large depth-width ratios, the reverse is true. The value of the critical depth-width ratios for which both formulas yield the same bearing value is a function of the angle of internal friction of the soil; the larger the value of the angle of internal friction, the larger is the value of the corresponding depth-width ratio. R. E. Fadum, USA

998. E. de Beer, Settlement records on bridges founded on sand, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 111–121 (1948).

In foundations on thick sand layers, the difficulty in taking undisturbed samples, especially underneath the ground-water table, had led to the use of sounding methods for getting the necessary data to compute the settlement of the buildings.

In order to control these methods, investigations were made in Belgium of 8 bridges where the settlements were regularly measured since the beginning of the work, and were compared with the computed settlements.

The results show a fair agreement between the computed and the real settlements, and lead to the conclusion that for sand layers the settlements based on sounding tests can be considered an upper bound of the settlements that can be expected.

Aurel A. Beleş, Rumania

999. Takeo Mogami, On the law of friction of sand, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 1, 51–54 (1948).

Measurements are reported of a decrease in the effective angle of friction in sand during the initial stages of a shear-box test. A theory is offered to explain the decrease in terms of a temporary shear layer of finite breadth due to dilatancy.

Dr. Rodney Hill, England

1000. Haruo Matuo, A test of the internal angle of soil under ground water (in Japanese), J. civ. Eng. Soc. Japan, special issue, no. 1–2, 67–68 (June 1947).

Soils were poured into a box 40 × 56 × 40 cm filled with water, and the side pressure was measured by a condenser pressure gage. The upper surface level of the water was varied above the soil surface. The author finds that the side pressure is

$$p = c_1 w' h_1 + c_2 w_0 h_2,$$

where c_1, c_2 are constants, w' is the weight of unit volume of soil in water, w_0 the weight of unit volume of water, h_1 the distance of the center of the gage from the surface of soil, h_2 the distance of the center of the gage from the surface of water.

T. Mogami, Japan

1001. **Yu. P. Vinogradov and P. P. Kufarev, On a problem of filtration** (in Russian), Prikl. Mat. Mekh. 12, 181–198 (1948).

The following problem is considered. In an incompressible fluid, which fills some two-dimensional region G_t that varies with the time t, there exists at a point $z = 0$ an overflow of magnitude $2\pi q(t)$ which changes with time. The velocity potential is assumed to be constant on the boundary C_t of G_t during some interval of time. It is required to determine the nature of the motion and the shape of the region G_t if the region G_0 occupied by the fluid at time $t = 0$ is given. On the basis of the results of P. J. Polubarinova-Kochina [Prikl. Mat. Mekh. 9, 79–90 (1945); C. R. Acad. Sci. URSS 47, 250–254 (1945)], and L. A. Galin [C. R. Acad. Sci. URSS 47, 246–249 (1945)] this problem is reduced to the solution of a system of integrodifferential equations. A study of these equations permits the authors to draw certain conclusions on the existence and nature of the solution of a boundary-value problem to which the present problem had been reduced by Polubarinova-Kochina, and to develop two methods for the solution of the problem.

Courtesy of Mathematical Reviews H. P. Thielman, USA

1002. **E. Reinius, The influence of the regulation of a water current on the water table of adjacent territory** (in Swedish), Tekn. Tidskr. 77, 83–88 (Jan. 1947).

The author points out the inadequacy of Forschheimer's well-known approximate method for the solution of ground-water flow problems of the type indicated in the title. Then he discusses, in connection with specific examples, the possibilities for a practically useful solution. For the steady flow of ground water with a "free" water-table surface toward a river bed, under the assumption of rigidity of the permeable stratum, he obtains the stream lines by the usual graphic construction of isometric nets. The transient case (nonsteady flow) and the influence of the deformability of the permeable material are discussed with the aid of rough estimates. P. Neményi, USA

Geophysics, Meteorology, Oceanography

(See also Rev. 1002)

1003. **M. V. Wilkes, Oscillations of the earth's atmosphere,** Cambridge, University Press, 1949, ix + 74 pp., 23 figs. Cloth, 8.8 × 5.7 in., $3.

This well-documented monograph summarizes and evaluates the work done up to 1948 on the subject of tidal oscillations in the atmosphere. The observations of atmospheric tides are summarized; techniques used in the theoretical solutions to the problem are outlined; and the implications of these results for the structure and temperature distribution in the upper atmosphere are carefully demonstrated.

The author reaffirms the validity of the resonance theory of both the solar and the lunar tides, showing it to be in its major features consistent with existing observations, including the most modern ones made by V-2 rockets. It is demonstrated how the predominant solar 12-hour period can quite conceivably be amplified to its observed high ratio to the lunar 12-hour period. This is done by first showing that the component of the solar force exciting this mode is of the proper magnitude, and second that a corresponding little-damped free period in the atmosphere exists. This existence of this free period requires a low-temperature layer around the height of 80 km, which is indeed observationally verifiable. This free period is further checked by the theory and observation of pulse propagation.

While this booklet contains no results which are essentially new, it will be of value to researchers in and students of the upper atmosphere and of atmospheric tides. To those studying the meteorology of the troposphere it will be of indirect value as restricted to background. Joanne Starr Malkus, USA

1004. **L. Egersdorfer, Energy balance as a component of weather forecasting,** Z. Met. 1, 229–235 (May-June 1947).

The mathematical form of a weather forecast is in terms of rate reactions, as exemplified by the Euler equation of motion. The author points out that the fundamental hydrothermodynamic equation of Ertel possesses this form and quality. The left side of this equation contains expressions for the density of the kinetic, gravitational, and internal energies, while the right side contains expressions for the density of surface work, the divergence of the turbulent heat current, the energy densities of the absorbed and emitted long-wave radiation, and the heats of condensation and evaporation. The respective contributions of these energy forms are estimated, and it is found that the economy of the atmosphere is such that in 10 hr per unit mass 1 to 2% of the available energy is transformed.

In conclusion, the different types of forecasts are listed, and a critical analysis is given of the measurements of those meteorological elements which must be expressed in the form of energies in order to make the Ertel equation practically useful.

Hans F. Winterkorn, USA

1005. **T. Hoshino and S. Syōno, Forecasting of the wind at higher levels** (in English), Geophys. Mag. Tokyo 20, 12–20 (Nov. 1948)

Assuming that the variation of upper westerlies is caused by meridional displacement, and using the theorem of the conservation of absolute circulation, the authors propose a method for forecasting the intensity of upper westerlies. The method is based on the idea that the meridional displacement of the upper air is caused by the subsidence of upper isobaric surfaces due to the outburst from the cold anticyclone. The method was tested by comparing synoptic charts and aerological observations. The result is satisfactory. K. Terada, Japan

1006. **Hans Ertel, A method of approximate prediction of the displacements of air masses** (in German), S.-B. Deutsch. Akad. Wiss. Berlin, Math.-Nat. Kl. 1948, no. 3, 23 pp. (1948).

A system of displacement equations is developed whose solution shows the air mass arriving at a given spot at a future time. Since the equations and their solution take into account only the first-order terms of the Taylor series by which the quantities have to be expressed, the solution represents a first approximation. The air mass is assumed to move horizontally along the earth's surface. Even with this assumption the required terms involving the horizontal variations of the horizontal wind-velocity components cannot be determined with sufficient accuracy in practice. Therefore a process of successive approximation is given which permits a practical solution and which converges, provided that the forecast interval is chosen sufficiently short, in two examples half a day

and a day, respectively, depending on the spatial variation of the wind field. In practice this successive approximation can be done graphically in a simple manner.

Courtesy of Mathematical Reviews B. Haurwitz, USA

1007. G. Skeib, On thermic convection (in German), Z. Met. 1, 178–183 (Mar. 1947).

Thermal convection is conceived as covering all those small-scale streaming forms of air for which radiation (either irradiation or emission) is the direct source of energy; excluded are systems in which condensation represents an additional source of energy.

Common concepts of convection, based on the picture of the warm-air tube or the rising warm-air pocket, are considered insufficient, and microconvection is given special consideration. The latter is responsible for the slow, stationary labilization of the lower air layer (which had become stable through nocturnal radiation) and initiates and augments macroconvection.

The often observed short-time periodic variations of the meteorologic elements in the lower layers suggest such periodicity in space and time also for microconvection. In order to avoid confusing of microconvection with the still very imperfectly defined concepts of fully developed dynamic turbulence, the considerations and mathematical formulations of the paper are restricted to days of moderate horizontal air movement. The quantitatively representable facts of the rise of a hot-air pocket are treated first, followed by the concept of a thermally unstable current field.

From the formation of both concepts, conclusions are drawn for the scheme of thermal microconvection which is sharply defined against the phenomena of dynamic turbulence.

Hans F. Winterkorn, USA

Lubrication; Bearings; Wear

1008. Daikaku Manabe, On the motion of shaft-center in bearings, 1st report (in Japanese), Trans. Soc. mech. Engrs. Japan 15, no. 50, III, 5–12 (1949).

Starting from the fundamental Reynolds equation of lubrication, the author analyzes the motion of the shaft center in a bearing. The analysis is too complex to be worked out in general, and approximate solutions are obtained for a few simple cases. These are: (1) shaft is stationary, and suffers a radial shock; (2) shaft is revolving with constant speed; (3) unbalanced force is acting on the shaft; (4) shaft begins to rotate from stationary position; (5) application to thrust-plane bearing. Norimune Soda, Japan

Marine Engineering Problems

(See also Revs. 923, 926, 965, 966)

1009. Kenneth S. M. Davidson and Anthony Suarez, Tests of twenty related models of V-bottom motor boats, David Taylor Model Basin Rep. no. 170, 103 pp. (Dec. 1948).

Designers of V-bottom motorboats can find in this report a complete and systematized set of charts, including test data, for a set of related models. These cover normal and extreme variations in beam-draft ratio and displacement-length ratio. The models were made 40 in. in length, and they are shown in comparison, photographically. Lines are given, with table of offsets, for the parent form together with tables of multipliers for different models, and dimensions of beam and draft.

The results are arranged in groups of charts. The contours of resistances are grouped for normal displacement, also for 10% excess displacement, and for 20% excess displacement. Variations are shown for static trim by the stern, at selected, constant speed-length ratios and for the different beam-draft ratios. In contour charts of running trim a preliminary diagram indicates forces, tow-point, and shaft angle. Next appear charts for longitudinal centers of gravity, with relation to static trim by the stern, which later is thus obtained much more simply than by equations. Then, there follow the wetted-surface charts, to permit making expansions to full size by treating the friction and residual components independently, since there is a large variation in wetted surface with speed in the case of V-bottom forms. The final charts are for porpoising, or longitudinal instability with increase of speed. Limiting speed-length ratios are shown on the charts.

At the beginning of the report, a comprehensive, over-all view of resistances is shown. This chart indicates: (1) the major importance of static trim, (2) the pronounced importance of displacement-length ratio, (3) the lesser importance of beam-draft ratio and excess displacement.

Instructions are given, at the beginning, together with a worked, numerical example. This leads to an estimate of power. To make a preliminary estimate, on the basis of Series 50 models (used in this report) it is usually necessary, first, to transform given values of beam/length and longitudinal-center-of-gravity/length into equivalent values of beam/draft and static-trim-by-the-stern. Neither of the latter has the simple, straightforward significance, here, that either has for steamship forms.

The report is a revision of a former report, published by the David Taylor Model Basin, in October 1941, for tests at the Stevens Experimental Towing Tank. The revision adds charts for running trim, the drawing of the parent form, and conversion factors. Eastman Smith, USA

1010. R. Spronck, Research towards a better utilization of the propulsive power of river boats (in French), Hommage Faculty appl. Sci. Univ. Liége, 240–253 (1947).

A general theory of boat performance is given and is presented in ABAC form for a particular series of screw propellers of which the efficiencies have been experimentally determined. The author then gives numerical examples to show how improvements in propulsive efficiency may be effected in the design or modification of small boats. The major emphasis is on selection of the optimum propeller conditions. R. Smelt, USA